Application of Integrals

Question1

Let A_1 be the area of the region bounded by the curves $y = \sin x$, $y = \cos x$ and y-axis in the first quadrant. Also, let A_2 be the area of the region bounded by the curves $y = \sin x$, $y = \cos x$, x-axis and $x = \frac{\pi}{2}$ in the first quadrant. Then,
[26 Feb 2021 Shift 2]

Options:

A. $A_1 : A_2 = 1 : \sqrt{2}$ and $A_1 + A_2 = 1$

B. $A_1 = A_2$ and $A_1 + A_2 = \text{sqrt}\,2$

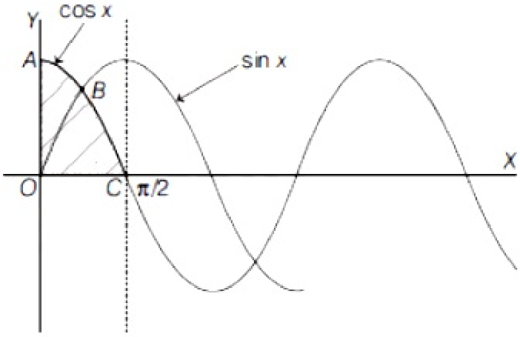
C. $2A_1 = A_2$ and $A_1 + A_2 = 1 + \sqrt{2}$

D. $A_1 : A_2 = 1 : 2$ and $A_1 + A_2 = 1$

Answer: A

Solution:

Solution:



A_1 is the area of region OAB.
A_2 is the area of region OBC.

Coordinate of B is $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$

Now, $A_1 = \int_0^{\frac{\pi}{4}} (\cos x - \sin x) dx$

$= [\sin x + \cos x]_0^{\pi/4}$

$= \left(\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right) - (0 + 1) = \sqrt{2} - 1$

$A_2 = \int_0^{\frac{\pi}{4}} \sin x\, dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x\, dx$

$= [\sin x + \cos x]_0^{\pi/4}$

$= \left(\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right) - (0 + 1) = \sqrt{2} - 1$

$A_2 = \int_0^{\frac{\pi}{4}} \sin x \, dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x \, dx$

$A_2 = [-\cos x]_0^{\frac{\pi}{4}} + [\sin x]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \sqrt{2}(\sqrt{2} - 1)$

Now, $A_1 : A_2 = (\sqrt{2} - 1) : \sqrt{2}(\sqrt{2} - 1)$

$A_1 : A_2 = 1 : \sqrt{2}$

and $A_1 + A_2 = (\sqrt{2} - 1) + \sqrt{2}(\sqrt{2} - 1) = (\sqrt{2} - 1)(\sqrt{2} + 1)$

$A_1 + A_2 = 2 - 1 = 1$

Therefore,

and $A_1 : A_2 = 1 : \sqrt{2}$,

$A_1 + A_2 = 1$

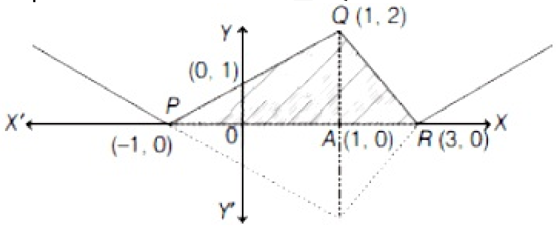
Question2

The area bounded by the lines $y = ||x - 1| - 2|$ is ....
[26 Feb 2021 Shift 1]

Answer: 4

Solution:

Solution:
Given, $y = ||x - 1| - 2|$
Required area is area of $\triangle PQR$.



Area $= \frac{1}{2} \times (\text{Base}) \times (\text{Height})$

$= \frac{1}{2} \times (PR) \times (AQ)$

$= \frac{1}{2} \times 4 \times 2 = 4$

Since, only one curve is given, here assume the area bounded by X-axis. Then, the area will be 4 sq unit.

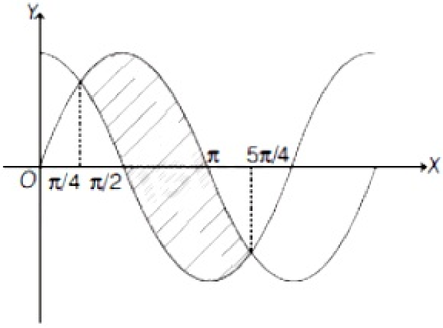
Question3

The graph of sine and cosine functions, intersect each other at a number of points and between two consecutive points of intersection, the two graphs enclose the same area A. Then A^4 is equal to ....
[25 Feb 2021 Shift 1]

Answer: 64

Solution:

Solution:



Required area of shaded region

$A = \int_{\pi/4}^{5\pi/4} \{\sin x - \cos x) d x$

$= [-\cos x - \sin x]_{\pi/4}^{5\pi/4}$

$= -\left[\left(\cos\frac{5\pi}{4} + \sin\frac{5\pi}{4}\right) - \left(\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\right)\right]$

$= -\left[\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right]$

$\therefore A = \frac{4}{\sqrt{2}} = 2\sqrt{2}$

$\Rightarrow A^4 = (2\sqrt{2})^4 = 64$

Question4

The area of the region $R = \{(x, y) : 5x^2 \le y \le 2x^2 + 9\}$ is [24 Feb 2021 Shift 2]

Options:

A. $11\sqrt{3}$ square units

B. $12\sqrt{3}$ square units

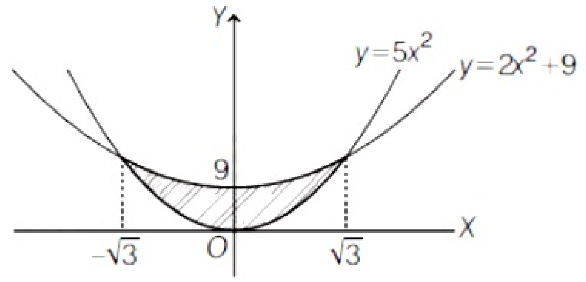
C. $9\sqrt{3}$ square units

D. $6\sqrt{3}$ square units

Answer: B

Solution:

Solution:
Given, $R = \{(x, y) : 5x^2 \le y \le 2x^2 + 9\}$
Here, we have two curves $y = 5x^2$ and $y = 2x^2 + 9$, point of intersection of both curves is find by solving both equations i.e.
$5x^2 = 2x^2 + 9$
$\Rightarrow x^2 = 3 \Rightarrow x = \pm\sqrt{3}$



$\therefore$ Area $= \int_{-\sqrt{3}}^{\sqrt{3}} (2x^2 + 9 - 5x^2)\,dx$

$= 2\int_{0}^{\sqrt{3}} (9 - 3x^2)\,dx$

$= 2[9x - x^3]_0^{\sqrt{3}}$

$= 2[9\sqrt{3} - 3\sqrt{3}]$

$= 12\sqrt{3}$ sq units

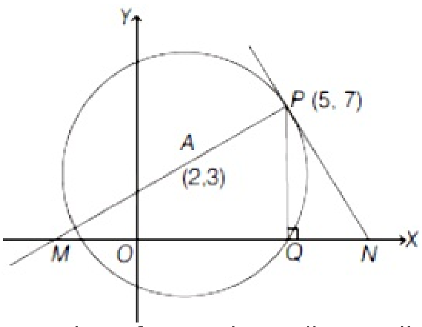
Question5

If the area of the triangle formed by the positive x-axis, the normal and the tangent to the circle $(x-2)^2 + (y-3)^2 = 25$ at the point (5, 7) is A, then 24A is equal to ....
[24 Feb 2021 Shift 2]

Answer: 1225

Solution:

Solution:
Given, circle $(x-2)^2 + (y-3)^2 = 5^2$
$c = (2, 3)$
$r = 5$



Equation of normal at P (i.e. PA line)

$\Rightarrow (y-7) = \left(\frac{7-3}{5-2}\right)(x-5)$

$\Rightarrow 3y - 21 = 4x - 20$

$\Rightarrow 4x - 3y + 1 = 0$

Therefore, $M = \left(\frac{-1}{4}, 0\right)$ [Put y = 0 in above equation]

Now, equation of tangent at P.

$y - 7 = \frac{-3}{4}(x-5)$ $\left[\because \text{ slope of PN} = \frac{-1}{\text{Slope of PA}}\right]$

$\Rightarrow 4y - 28 = -3x + 15$

$\Rightarrow 3x + 4y = 43$

Therefore, $N = \left(\frac{43}{3}, 0\right)$ [Put y = 0 in above equation]

$= \frac{1}{2} \times \left(\frac{43}{3} + \frac{1}{4} \right) \times 7$

$= \frac{1}{2} \times \frac{175}{12} \times 7$

$\therefore\ 24A = 24 \times \frac{1}{2} \times \frac{175}{12} \times 7 = 1225$

But this question is wrong as in question. It is mentioned that the triangle is formed with the positive X -axis which contradicts the solution.

Question6

The area (in sq. units) of the part of the circle $x^2 + y^2 = 36$, which is outside the parabola $y^2 = 9x$, is :
24 Feb 2021 Shift 1

Options:

A. $24\pi + 3\sqrt{3}$

B. $12\pi - 3\sqrt{3}$

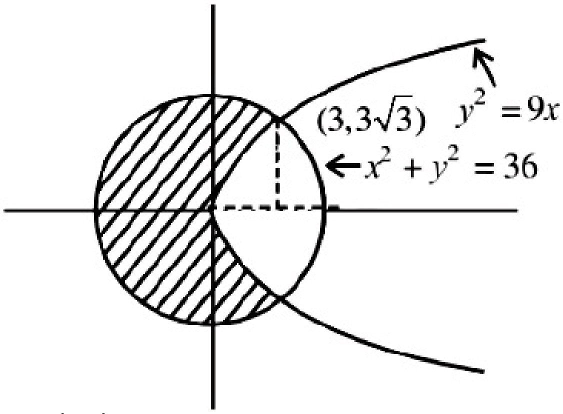
C. $24\pi - 3\sqrt{3}$

D. $12\pi + 3\sqrt{3}$

Answer: C

Solution:

Solution:



Required area

$= \pi \times (6)^2 - 2\int_0^3 \sqrt{9x}\,dx - 2\int_3^6 \sqrt{36 - x^2}\,dx$

$= 36\pi - 12\sqrt{3} - 2\left(\frac{x}{2}\sqrt{36 - x^2} + 18\sin^{-1}\frac{x}{6} \right)_3^6$

$= 36\pi - 12\sqrt{3} - 2\left(9\pi - 3\pi - \frac{9\sqrt{3}}{2} \right)$

$= 24\pi - 3\sqrt{3}$

Question7

Let $g(x) = \int^{x} f(t)\,dt$, where f is continuous function in [0, 3] such that

$\frac{1}{3} \le f(t) \le 1$ for all $t \in [0, 1]$ and $0 \le f(t) \le \frac{1}{2}$ for all $t \in (1, 3]$. The largest possible interval in which g(3) lies is
[18 Mar 2021 Shift 2]

Options:

A. $\left[-1, -\frac{1}{2}\right]$

B. $\left[-\frac{3}{2}, -1\right]$

C. $\left[\frac{1}{3}, 2\right]$

D. [1, 3]

Answer: C

Solution:

Solution:

Given, $g(x) = \int_0^x f(t)dt$

$\therefore\ g(3) = \int_0^3 f(t)dt = \int_0^1 f(t)dt + \int_1^3 f(t)dt$

$\Rightarrow \int_0^1 \frac{1}{3}dt + \int_1^3 0 \cdot dt \le g(3) \le \int_0^1 1dt + \int_1^3 \frac{1}{2}dt$

$\Rightarrow \frac{1}{3} \le g(3) \le 1 + 1$

$\Rightarrow \frac{1}{3} \le g(3) \le 2$

Question8

The area bounded by the curve $4y^2 = x^2(4 - x)(x - 2)$ is equal to
[18 Mar 2021 Shift 2]

Options:

A. $\frac{\pi}{8}$

B. $\frac{3\pi}{8}$

C. $\frac{3\pi}{2}$

D. $\frac{\pi}{16}$

Answer: C

Solution:

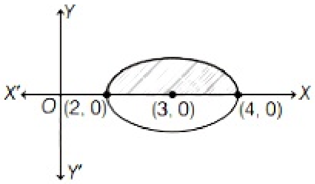
$\Rightarrow \sqrt{4y^2} = \sqrt{x^2(4-x)(x-2)}$

$\Rightarrow |y| = \frac{|x|}{2}\sqrt{(4-x)(x-2)}$

$$y = \begin{cases} \frac{x}{2}\sqrt{(4-x)(x-2)} & x > 0 \\ -\frac{x}{2}\sqrt{(4-x)(x-2)} & x < 0. \end{cases}$$

Let

$y_1 = \frac{x}{2}\sqrt{(4-x)(x-2)} \Rightarrow y_2 = -\frac{x}{2}\sqrt{(4-x)(x-2)}$



and domain x in[2, 4]

$[\because (4-x)(x-2) \geq 0 \Rightarrow (x-2)(x-4) \leq 0 \Rightarrow 2 \leq x \leq 4]$

$\therefore$ Required area $= \int_2^4 (y_1 - y_2)dx$

$= \int_2^4 x\sqrt{(4-x)(x-2)}dx \ldots$(i)

Using property, $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

From Eq. (i),

Area $= \int_2^4 (6-x)\sqrt{(4-x)(x-2)}dx \ldots$(ii)

From Eqs. (i) and (ii), $2A = 6\int_2^4 \sqrt{(4-x)(x-2)}dx$

$\Rightarrow A = 3\int_2^4 \sqrt{1-(x-3)^2}dx$

$A = 3\left[\frac{x-3}{2}\sqrt{1-(x-3)^2} + \frac{1}{2}\sin^{-1}(x-3)\right]_2^4$

$A = \frac{3}{2}\left[\frac{0+\pi}{2} - \frac{0+\pi}{2}\right] \Rightarrow A = \frac{3\pi}{2}$

$\Rightarrow A = 3 \cdot \frac{\pi}{2} \cdot (1)^2 = \frac{3\pi}{2}$

Question9

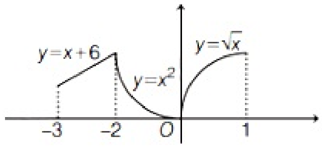
Let $f : [-3, 1] \rightarrow R$ be given as $f(x) = \left\{ \begin{array}{ll} \min\{(x+6), x^2\} & -3 \leq x \leq 0 \\ \max\{\sqrt{x}, x^2\} & 0 \leq x \leq 1 \end{array} \right\}$

If the area bounded by $y = f(x)$ and x-axis is A, then the value of 6A is equal to
[17 Mar 2021 Shift 2]

Answer: 41

Solution:

Solution:



A = Area bounded by y = f(x) and X-axis.

$= \int_{-3}^{-2}(x+6)dx + \int_{-2}^{0}x^2dx + \int_{0}^{1}\sqrt{x}dx$

$= \left[\frac{x^2}{2}\right]_{-3}^{-2} + 6[x]_{-3}^{-2} + \left[\frac{x^3}{3}\right]_{-2}^{0} + \left[\frac{2}{3}x^{\frac{3}{2}}\right]_{0}^{1} = \frac{41}{6}$

$\therefore 6A = 6 \times \frac{41}{6}$

$\Rightarrow 6A = 41$

Question10

If the area of the bounded region

$R = \left\{ (x, y) : \max\{0, \log_e x\} \le y \le 2^x, \frac{1}{2} \le x \le 2 \right\}$

is, $\alpha(\log_e 2)^{-1} + \beta(\log_e 2) + \gamma$, then the value of $(\alpha + \beta - 2\gamma)^2$ is equal to :
[27 Jul 2021 Shift 1]

Options:

A. 8

B. 2

C. 4

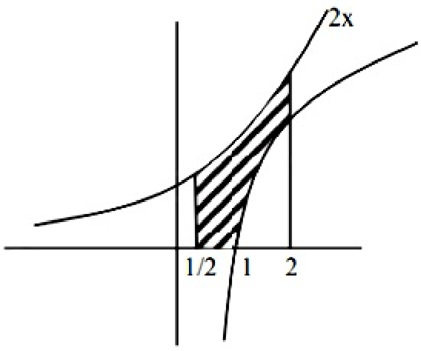
D. 1

Answer: B

Solution:

Solution:

$R = \left\{ (x, y) : \max\{0, \log_e x\} \le y \le 2^x, \frac{1}{2} \le x \le 2 \right\}$



$\int_{\frac{1}{2}}^{2} 2^x dx - \int_{1}^{2} \ln x dx$

$\Rightarrow \left[\frac{2^x}{\ln 2}\right]_{1/2}^{2} - [x\ln x - x]_1^2$

$\Rightarrow \frac{(2^2) - 2^{1/2}}{\log 2} - (2\ln 2 - 1)$

$\Rightarrow \frac{(2^2 - \sqrt{2})}{\log_e 2} - 2\ln 2 + 1$
$\therefore \alpha = 2^2 - \sqrt{2}, \beta = -2, \gamma = 1$
$\Rightarrow (\alpha + \beta + 2\gamma)^2$
$\Rightarrow (2^2 - \sqrt{2} - 2 - 2)^2$
$\Rightarrow (\sqrt{2})^2 = 2$

Question11

The area of the region bounded by $y - x = 2$ and $x^2 = y$ is equal to :- [27 Jul 2021 Shift 2]

Options:

A. $\frac{16}{3}$

B. $\frac{2}{3}$

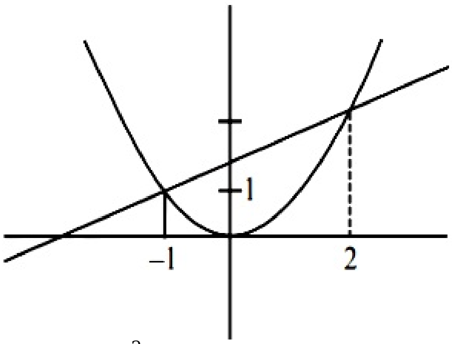
C. $\frac{9}{2}$

D. $\frac{4}{3}$

Answer: C

Solution:

Solution:



$y - x = 2, x^2 = y$
Now, $x^2 = 2 + x$
$\Rightarrow x^2 - x - 2 = 0$
$\Rightarrow (x + 1)(x - 2) = 0$
Area $= \int_{-1}^{2} (2 + x - x^2)$
$= \left| 2x + \frac{x^2}{2} - \frac{x^3}{3} \right|_{-1}^{2}$
$= \left(4 + 2 - \frac{8}{3}\right) - \left(-2 + \frac{1}{2} + \frac{1}{3}\right)$
$= 6 - 3 + 2 - \frac{1}{2} = \frac{9}{2}$

Question12

$\{(x, y) \in R \times R \mid x \geq 0, 2x^2 \leq y \leq 4 - 2x\}$ is :
[25 Jul 2021 Shift 1]

Options:

A. $\frac{8}{3}$

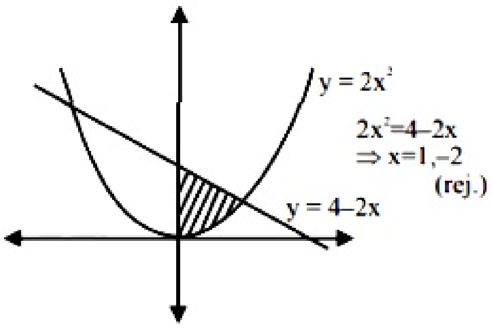
B. $\frac{17}{3}$

C. $\frac{13}{3}$

D. $\frac{7}{3}$

Answer: D

Solution:



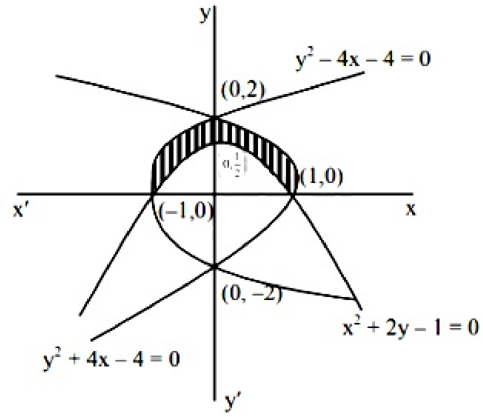
Required area $= \int_0^1 (4 - 2x - 2x^2)dx = 4x - x^2 - \frac{2x^3}{3}\Big|_0^1 = 4 - 1 - \frac{2}{3} = \frac{7}{3}$

Question13

The area (in sq. units) of the region bounded by the curves $x^2 + 2y - 1 = 0$, $y^2 + 4x - 4 = 0$ and $y^2 - 4x - 4 = 0$in the upper half plane is ________.
[22 Jul 2021 Shift 2]

Answer: 2

Solution:



Required Area (shaded)

$= 2\left[\int_0^2 \left(\frac{4-y^2}{4}\right) dy - \int_0^1 \left(\frac{1-x^2}{2}\right) dx\right]$

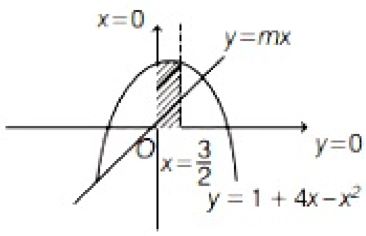
$= 2\left[\frac{4}{3} - \frac{1}{3}\right] = (2)$

Question14

If the line $y = mx$ bisects the area enclosed by the lines $x = 0$ and $y = 0$, $x = \frac{3}{2}$ and the curve $y = 1 + 4x - x^2$, then 12m is equal to [31 Aug 2021 Shift 2]

Answer: 26

Solution:



According to the question,

$\frac{1}{2}\int_0^{\frac{3}{2}} (1 + 4x - x^2) dx = \int_0^{\frac{3}{2}} mx dx$

$\Rightarrow \frac{1}{2}\left[\left(x + 2x^2 - \frac{x^3}{3}\right)\right]_0^{\frac{3}{2}} = \frac{m}{2}[x]_0^{\frac{3}{2}}$

$\Rightarrow \frac{3}{2} + \frac{9}{2} - \frac{9}{8} = \frac{9m}{4}$

$\Rightarrow m = \frac{39}{18}$

$\Rightarrow 12m = 26$

Question15

The area of the region bounded by the parabola $(y - 2)^2 = (x - 1)$, the

[27 Aug 2021 Shift 2]

Options:

A. 9

B. 10

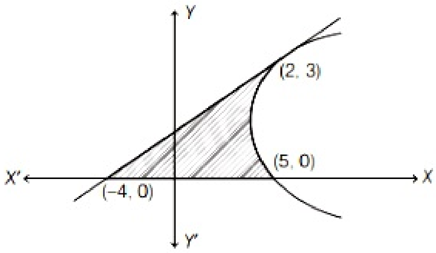
C. 4

D. 6

Answer: A

Solution:

Solution:
Given parabola
$(y-2)^2 = (x-1)$
Since, Ordinate = y = 3
Then, x = 2
Point on parabola (2, 3)
Differentiating Eq. (i) w.r.t. x, we get
$2(y-2)\frac{dy}{dx} = 1$



$\frac{dy}{dx} = \frac{1}{2(y-2)}$
At (2, 3)
$\frac{dy}{dx} = \frac{1}{2}$
Equation of tangent at (2, 3)
$y - 3 = \frac{1}{2}(x-2)$
or x − 2y + 4 = 0
Intersection point of parabola on X-axis is
y = 0, x = 5 i.e (5, 0)
Intersection point of tangent and X-axis
y = 0, x = − 4 i.e (− 4, 0)
Area of shaded region $= \int_0^3 [(y-2)^2 + 1 - (2y-4)]dy$
$= \int_0^3 (y^2 - 6y + 9)dy$
$= \left(\frac{y^3}{3} - 3y^2 + 9y\right)_0^3 = 9$ sq. unit.

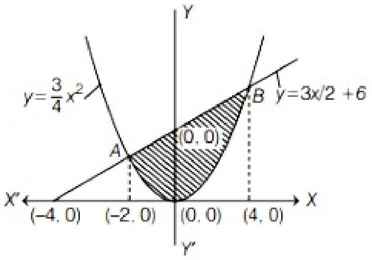
Question16

The area of the region $S = \{(x, y) : 3x^2 \le 4y \le 6x + 24\}$ is [26 Aug 2021 Shift 1]

Answer: 27

Solution:

Solution:



We have, $y = \left(\frac{3}{4}\right)x^2$ and $y = \left(\frac{3x}{2}\right) + 6$

$\Rightarrow \frac{3x^2}{4} = \frac{3x}{2} + 6$

$\Rightarrow 3x^2 = 6x + 24$

$\Rightarrow x^2 - 2x - 8 = 0$

$\Rightarrow (x - 4)(x + 2) = 0$

$\Rightarrow x = -2, 4$

$\Rightarrow y = 3, 12$

$A(-2, 3)$ and $B(4, 12)$

Required area $= \int_{-2}^{4} \left(\frac{3x}{2} + 6\right) - \left(\frac{3x^2}{4}\right) dx$

$= \left[\frac{3x^2}{4} + 6x - \frac{x^3}{4}\right]_{-2}^{4}$

$= [(12 + 24 - 16) - (3 - 12 + 2)]$

$= (20 + 7) = 27$ sq units

Question17

Let a and b respectively be the points of local maximum and local minimum of the function $f(x) = 2x^3 - 3x^2 - 12x$. If A is the total area of the region bounded by $y = f(x)$, the X- axis and the lines $x = a$ and $x = b$, then 4A is equal to [26 Aug 2021 Shift 2]

Answer: 114

Solution:

We have, $f(x) = 2x^3 - 3x^2 - 12x$

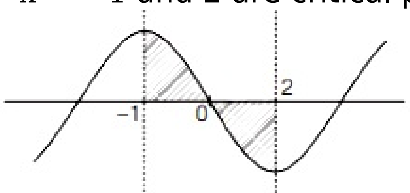
$\therefore f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2)$

$= 6(x + 1)(x - 2)$

$f'(x) = 0$

$\Rightarrow x = -1$ and 2

$\therefore x = -1$ and 2 are critical points



$\therefore a = -1$ and $b = 2$

Now, required area, $A_0 = \int_{-1}^{0} f(x) dx + \int_{0}^{2} - f(x) dx$

$$\int_{-1}^{0}(2x^3-3x^2-12x)dx+\int_{0}^{2}(12x+3x^2-2x^3)dx$$
$$=\left[\frac{x^4}{2}-x^3-6x^2\right]_{-1}^{0}+\left[6x^2+x^3-\frac{x^4}{2}\right]_{0}^{2}=\frac{114}{4}$$
$\therefore 4A=114$

Question18

The area, enclosed by the curves $y=\sin x+\cos x$ and $y=|\cos x-\sin x|$ and the lines $x=0$, $x=\frac{\pi}{2}$, is
[1 Sep 2021 Shift 2]

Options:

A. $2\sqrt{2}(\sqrt{2}-1)$

B. $2(\sqrt{2}+1)$

C. $4(\sqrt{2}-1)$

D. $2\sqrt{2}(\sqrt{2}+1)$

Answer: A

Solution:

Solution:

Area $=\int_0^{\frac{\pi}{2}}((\cos x+\sin x)-|\cos x-\sin x|)dx$

$=\int_0^{\frac{\pi}{4}}((\cos x+\sin x)-(\cos x-\sin x))dx+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}((\cos x+\sin x)-(\sin x-\cos x))dx$

$=2\int_0^{\frac{\pi}{4}}\sin x\,dx+2\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\cos x\,dx$

$=2\left(\frac{-1}{\sqrt{2}}+1\right)+2\left(1-\frac{1}{\sqrt{2}}\right)=2\sqrt{2}(\sqrt{2}-1)$

Question19

The area of the region, enclosed by the circle $x^2+y^2=2$ which is not common to the region bounded by the parabola $y^2=x$ and the straight line $y=x$, is:
[Jan. 7, 2020 (I)]

Options:

A. $(24\pi-1)$

B. $(6\pi-1)$

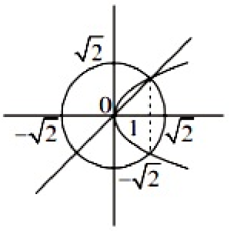
C. $(12\pi-1)$

D. $(12\pi - 1) / 6$

Answer: D

Solution:

Solution:
Total area – enclosed area between line and parabola



$$= 2\pi - \int_0^1 \sqrt{x} - x\,dx$$

$$= 2\pi - \left(\frac{2x^{3/2}}{3} - \frac{x^2}{2}\right)_0^1$$

$$= 2\pi - \left(\frac{2}{3} - \frac{1}{2}\right) = 2\pi - \left(\frac{1}{6}\right) = \frac{12\pi - 1}{6}$$

Question20

The area (in sq. units) of the region $\{(x, y) \in R^2 \mid 4x^2 \le y \le 8x + 12\}$ is: [Jan. 7, 2020 (II)]

Options:

A. $\frac{125}{3}$

B. $\frac{128}{3}$

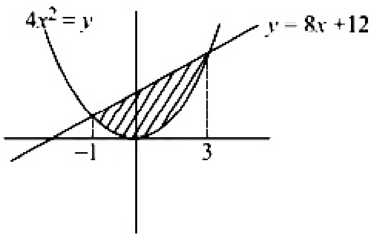
C. $\frac{124}{3}$

D. $\frac{127}{3}$

Answer: B

Solution:

Solution:



Given curves are
$4x^2 = y$ ......(i)
$y = 8x + 12$ .....(ii)
From eqns. (i) and (ii),

$\Rightarrow x^2 - 2x - 3 = 0$
$\Rightarrow x^2 - 3x + x - 3 = 0$
$\Rightarrow (x+1)(x-3) = 0$
$\Rightarrow x = -1, 3$
Required area bounded by curves is given by

$$A = \int_{-1}^{3} (8x + 12 - 4x^2) dx$$

$$A = \left[\frac{8x^2}{2} + 12x - \frac{4x^3}{3}\right]_{-1}^{3}$$

$$= (4(9) + 36 - 36) - \left(4 - 12 + \frac{4}{3}\right)$$

$$= 36 + 8 - \frac{4}{3} = 44 - \frac{4}{3} = \frac{132 - 4}{3} = \frac{128}{3}$$

Question21

For a > 0, let the curves $C_1 : y^2 = ax$ and $C_2 : x^2 = ay$ intersect at origin O and a point P. Let the line x = b(0 < b < a) intersect the chord OP and the x -axis at points Q and R, respectively. If the line x = b bisects the area bounded by the curves, C_1 and C_2, and the area of $\Delta OQR = \frac{1}{2}$, then 'a' satisfies the equation:
[Jan. 8, 2020 (I)]

Options:

A. $x^6 - 6x^3 + 4 = 0$

B. $x^6 - 12x^3 + 4 = 0$

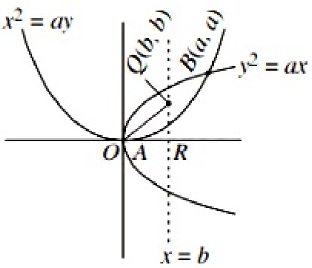
C. $x^6 + 6x^3 - 4 = 0$

D. $x^6 - 12x^3 - 4 = 0$

Answer: B

Solution:

Solution:
Given eqns. are, $x^2 = ay$ and $y^2 = ax$



After solving, we get $x = a, y = a$
Now, coordinates of B is (a, a) and A is (0,0)
Now, coordinates of Q is (b, b)

$$\frac{1}{2}b^2 = \frac{1}{2} \Rightarrow b = 1$$

Area bounded by curves and $x = 1$ is

$$\int_0^1 \left(\sqrt{a}x^{1/2} - \frac{x^2}{a}\right) dx = \frac{1}{2}\int_0^a \left(\sqrt{a}x^{1/2} - \frac{x^2}{a}\right) dx$$

$\Rightarrow \frac{2}{\ldots}\sqrt{\ldots} - \frac{1}{\ldots} = \frac{a^2}{\ldots}$

$\Rightarrow a^6 + 4a^3 + 4 = 16a^3$
$\Rightarrow a^6 - 12a^3 + 4 = 0$

Question22

The area (in sq. units) of the region $\{ (x, y) \in R^2 : x^2 \le y \le |3 - 2x|$, is: [Jan. 8, 2020 (II)]

Options:

A. $\frac{32}{3}$

B. $\frac{34}{3}$

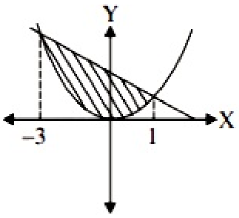
C. $\frac{29}{3}$

D. $\frac{31}{3}$

Answer: A

Solution:

Solution:
Point of intersection of $y = x^2$ and $y = -2x + 3$ is obtained by $x^2 + 2x - 3 = 0$



$\Rightarrow x = -3, 1$

So, required area $= \int_{-3}^{1} (\text{line} - \text{parabola})\,dz$

$= \int_{-3}^{1} (3 - 2x - x^2)\,dx = \left[3x - x^2 - \frac{x^3}{3}\right]_{-3}^{1}$

$= (3)4 - 2\left(\frac{1^2 - 3^2}{2}\right) - \left(\frac{1^3 + 3^3}{3}\right) = 12 + 8 - \frac{28}{3} = \frac{32}{3}$

Question23

Given:

$$f(x) = \begin{cases} x, & 0 \le x < \frac{1}{2} \\ \frac{1}{2}, & x = \frac{1}{2} \\ 1 - x, & \frac{1}{2} < x \le 1 \end{cases}$$

and g(x) = $\left(x - \frac{1}{2}\right)^2$, x ∈ R. Then the area (in sq. units) of the region bounded by the curves, y = f (x) and y = g(x) between the lines, 2x = 1 and 2x = $\sqrt{3}$, is:
[Jan. 9, 2020 (II)]

Options:

A. $\frac{1}{3} + \frac{\sqrt{3}}{4}$

B. $\frac{\sqrt{3}}{4} - \frac{1}{3}$

C. $\frac{1}{2} - \frac{\sqrt{3}}{4}$

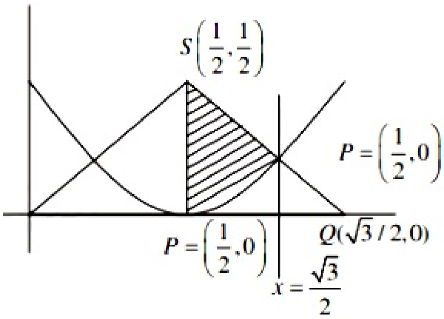
D. $\frac{1}{2} + \frac{\sqrt{3}}{4}$

Answer: B

Solution:

Solution:

Coordinates of $P\left(\frac{1}{2}, 0\right)$, $Q\left(\frac{\sqrt{3}}{2}, 0\right)$, $R\left(\frac{\sqrt{3}}{2}, 1 - \frac{\sqrt{3}}{2}\right)$ and $S\left(\frac{1}{2}, \frac{1}{2}\right)$



Required area = Area of trapezium PQRS

$$-\int_{1/2}^{\sqrt{3}/2}\left(x - \frac{1}{2}\right)^2 dx$$

$$= \frac{1}{2}\left(\frac{\sqrt{3} - 1}{2}\right)\left(\frac{1}{2} + 1 - \frac{\sqrt{3}}{2}\right) - \frac{1}{3}\left(\left(x - \frac{1}{2}\right)^3\right)_{1/2}^{\sqrt{3}/2}$$

$$= \frac{\sqrt{3}}{4} - \frac{1}{3}$$

Question24

The area (in sq. units) of the region A = { (x, y) : (x − 1)[x] ≤ y ≤ $2\sqrt{x}$, 0 ≤ x ≤ 2 }, where [t] denotes the greatest integer function, is:
[Sep. 05, 2020 (II)]

Options:

A. $\frac{8}{3}\sqrt{2} - \frac{1}{2}$

B. $\frac{4}{3}\sqrt{2} + 1$

C. $\frac{8}{3}\sqrt{2} - 1$

D. $\frac{4}{3}\sqrt{2} - \frac{1}{2}$

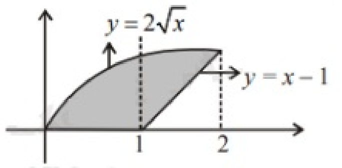
Answer: A

Solution:

Solution:

$[x] = 0$ when $x \in [0, 1)$ and $[x] = 1$ when $x \in [1, 2)$

$$y = \begin{cases} 0 & 0 \leq x < 1 \\ x - 1 & 1 \leq x < 2 \end{cases}$$



$$\therefore A = \int_0^2 2\sqrt{x}\,dx - \frac{1}{2}(1)(1)$$

$$= \frac{4x^{3/2}}{3}\bigg|_0^2 - \frac{1}{2} = \frac{8\sqrt{2}}{3} - \frac{1}{2}$$

Question25

The area (in sq. units) of the region

$$\left\{ (x, y) : 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, \frac{1}{2} \leq x \leq 2 \right\}$$

is

[Sep. 03, 2020 (I)]

Options:

A. $\frac{23}{16}$

B. $\frac{79}{24}$

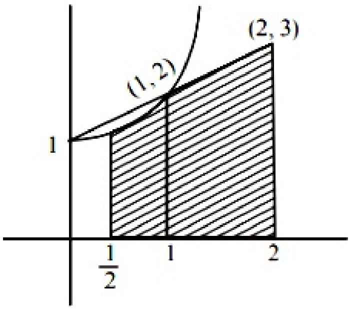
C. $\frac{79}{16}$

D. $\frac{23}{6}$

Answer: B

Solution:

Solution:



Required area $= \int_{\frac{1}{2}}^{1} (x^2+1)dx + \int_{1}^{2} (x+1)dx$

$= \left[\frac{x^3}{3}+x\right]_{\frac{1}{2}}^{1} + \left[\frac{x^2}{2}+x\right]_{1}^{2}$

$= \left[\frac{4}{3}-\frac{13}{24}\right] + \frac{5}{2} = \frac{79}{24}$

Question26

The area (in sq. units) of the region $A = \{(x, y): |x| + |y| \le 1, 2y^2 \ge |x|\}$ is:
[Sep. 06, 2020 (I)]

Options:

A. $\frac{1}{3}$

B. $\frac{7}{6}$

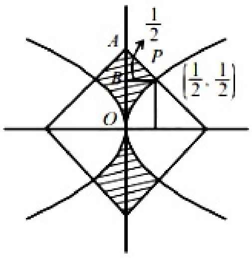
C. $\frac{1}{6}$

D. $\frac{5}{6}$

Answer: D

Solution:

Solution:



Required area $= 4\left[\int_{0}^{\frac{1}{2}} 2y^2 dy + \frac{1}{2}\text{area}(\Delta PAB)\right]$

$= 4\left[\frac{2}{3}[y^3]_0^{\frac{1}{2}} + \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right] = 4\left[\frac{2}{3}\times\frac{1}{8}+\frac{1}{8}\right]$

$= 4\times\frac{5}{24} = \frac{5}{6}$

Question27

The area (in sq. units) of the region enclosed by the curves $y = x^2 - 1$ and $y = 1 - x^2$ is equal to:
[Sep. 06, 2020 (II)]

Options:

A. $\frac{4}{3}$

B. $\frac{8}{3}$

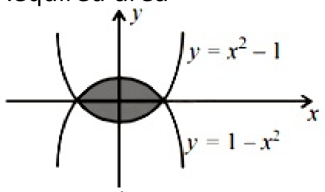
C. $\frac{7}{2}$

D. $\frac{16}{3}$

Answer: B

Solution:

Solution:
Required area



Area $= 2\int_0^1 ((1 - x^2) - (x^2 - 1))dx$

$= 4\int_0^1 (1 - x^2)dx$

$= 4\left(x - \frac{x^3}{3}\right)_0^1 = 4\left(1 - \frac{1}{3}\right) = 4 \cdot \frac{2}{3} = \frac{8}{3}$ sq. units

---

Question28

Consider a region $R = (x, y) \in \{R^2 : x^2 \le y \le 2x\}$. If a line $y = \alpha$ divides the area of region R into two equal parts, then which of the following is true?
[Sep.02,2020(II)]

Options:

A. $\alpha^3 - 6\alpha^2 + 16 = 0$

B. $3\alpha^2 - 8\alpha^{3/2} + 8 = 0$

C. $3\alpha^2 - 8\alpha + 8 = 0$

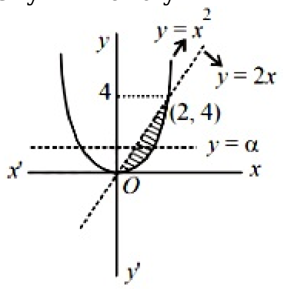
D. $\alpha^3 - 6\alpha^{3/2} - 16 = 0$

Answer: B

Solution:

Solution:

Let $y = x^2$ and $y = 2x$



According to question

$$\therefore \int_0^{\alpha}\left(\sqrt{y}-\frac{y}{2}\right)dy = \int_{\alpha}^{4}\left(\sqrt{y}-\frac{y}{2}\right)dy$$

$$\Rightarrow \left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{\alpha} - \left[\frac{y^2}{4}\right]_0^{\alpha} = \left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_{\alpha}^{4} - \left[\frac{y^2}{4}\right]_a^{4}$$

$$\Rightarrow \frac{2}{3}\alpha^{3/2} - \frac{\alpha^2}{4} = \frac{2}{3}(8-\alpha^{3/2}) - \frac{1}{4}(16-\alpha^2)$$

$$\Rightarrow \frac{4}{3}\alpha^{3/2} - \frac{\alpha^2}{2} = \frac{4}{3}$$

$$\Rightarrow 8\alpha^{3/2} - 3\alpha^2 = 8$$

$$\therefore 3\alpha^2 - 8\alpha^{3/2} + 8 = 0$$

Question29

The area (in sq. units) bounded by the parabola $y = x^2 - 1$, the tangent at the point (2,3) to it and the y -axis is:
[Jan. 9,2019 (I)]

Options:

A. $\frac{8}{3}$

B. $\frac{32}{3}$

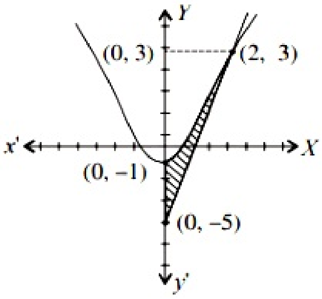
C. $\frac{56}{3}$

D. $\frac{14}{3}$

Answer: A

Solution:

Solution:



∵ Curve is given as :

$y = x^2 - 1$

$\Rightarrow \frac{dy}{dx} = 2x$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(2,3)} = 4$

∴ equation of tangent at (2,3)

$(y - 3) = 4(x - 2)$

$\Rightarrow y = 4x - 5$

but $x = 0$

$\Rightarrow y = -5$

Here the curve cuts Y-axis

∴ required area $= \frac{1}{4}\int_{-5}^{3}(y+5)dy - \int_{-1}^{3}\sqrt{y+1}\,dy$

$= \frac{1}{4}\left[\frac{y^2}{2} + 5y\right]_{-5}^{3} \frac{-2}{3}[(y+1)^{3/2}]_{-1}^{3}$

$= \frac{1}{4}\left[\frac{9}{2} + 15 - \frac{25}{2} + 25\right]$

$= -\frac{2}{3}[4^{3/2} - 0]$

$= \frac{32}{4} - \frac{16}{3} = \frac{8}{3}$ sq-units.

Question30

The area (in sq. units) of the region bounded by the parabola, $y = x^2 + 2$ and the lines, $y = x + 1$, $x = 0$ and $x = 3$ is :
[Jan. 12, 2019 (I)]

Options:

A. $\frac{15}{4}$

B. $\frac{21}{2}$

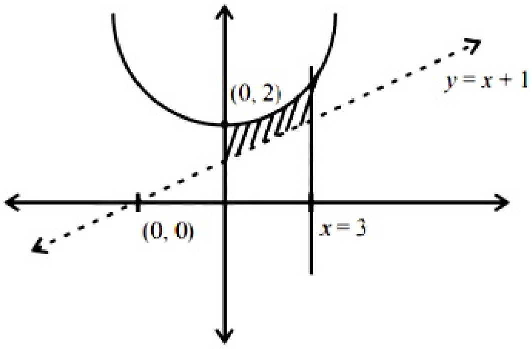
C. $\frac{17}{4}$

D. $\frac{15}{2}$

Answer: D

Solution:

Solution:



Area of the bounded region $\int_0^3 [(x^2+2)-(x+1)]dx$

$= \left[\frac{x^3}{3} - \frac{x^2}{2} + x\right]_0^3$

$= 9 - \frac{9}{2} + 3 = \frac{15}{2}$

Question31

The area (in sq. units) of the region bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$ is:
[Jan. 11, 2019 (I)]

Options:

A. $\frac{5}{4}$

B. $\frac{9}{8}$

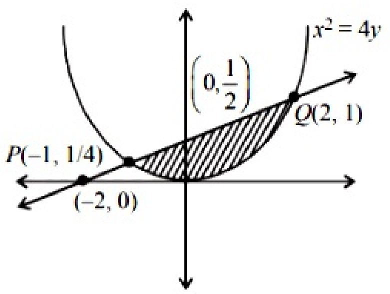
C. $\frac{7}{8}$

D. $\frac{3}{4}$

Answer: B

Solution:

Solution:



Let points of intersection of the curve and the line be P and Q

$x^2 = 4\left(\frac{x+2}{4}\right)$

$x^2 - x - 2 = 0$

$x = 2, -1$

Point are (2,1) and $\left(-1, \frac{1}{4}\right)$

Area $= \int_{-1}^{2}\left[\left(\frac{x+2}{4}\right)-\left(\frac{x^2}{4}\right)\right]dx = \left[\frac{x^2}{8}+\frac{1}{2}x-\frac{x^3}{12}\right]_{-1}^{2} = \left(\frac{1}{2}+1-\frac{2}{3}\right)-\left(\frac{1}{8}-\frac{1}{2}+\frac{1}{12}\right) = \frac{9}{8}$

Question32

The area (in sq. units) in the first quadrant bounded by the parabola, $y = x^2 + 1$, the tangent to it at the point (2,5) and the coordinate axes is: [Jan. 11, 2019 (II)]

Options:

A. $\frac{8}{3}$

B. $\frac{37}{24}$

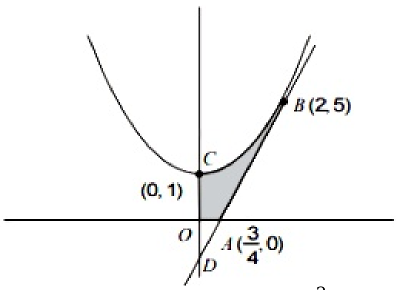
C. $\frac{187}{24}$

D. $\frac{14}{3}$

Answer: B

Solution:

Solution:



The equation of parabola $x^2 = y - 1$
The equation of tangent at (2,5) to parabola is
$y - 5 = \left(\frac{dy}{dx}\right)_{(2,5)}(x-2)$
$y - 5 = 4(x - 2)$
$4x - y = 3$
Then, the required area
$= \int_0^2 \{(x^2+1)-(4x-3)\}dx -$ Area of ΔAOD
$= \int_0^2 (x^2 - 4x + 4)dx - \frac{1}{2}\times\frac{3}{4}\times 3$
$= \left[\frac{(x-2)^3}{3}\right]_0^2 - \frac{9}{8} = \frac{37}{24}$

Question33

If the area enclosed between the curves $y = kx^2$ and $x = ky^2$, $(k > 0)$, is 1 square unit. Then k is: [Jan. 10, 2019 (I)]

Options:

A. $\frac{\sqrt{3}}{2}$

B. $\frac{1}{\sqrt{3}}$

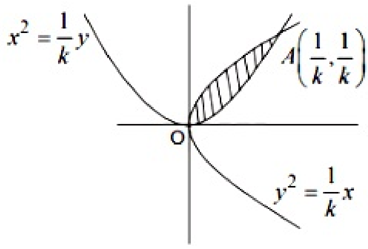
C. $\sqrt{3}$

D. $\frac{2}{\sqrt{3}}$

Answer: B

Solution:

Solution:



Two curves will intersect in the Ist quadrant at $A\left(\frac{1}{R}, \frac{1}{R}\right)$

$\because$ area of shaded region $= 1$

$\therefore \int_0^{\frac{1}{k}} \left(\frac{\sqrt{x}}{\sqrt{k}} - kx^2 \right) dx = 1$

$\Rightarrow \left(\frac{1}{\sqrt{k}} \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right)_0^{\frac{1}{k}} - \left(k \cdot \frac{x^3}{3} \right)_0^{\frac{1}{k}} = 1$

$\Rightarrow \frac{2}{3\sqrt{k}} \cdot \frac{1}{k^{\frac{3}{2}}} - \frac{k}{3k^3} = 1$

$\Rightarrow \frac{2}{3k^2} - \frac{1}{3k^2} = 1$

$\Rightarrow 3k^2 = 1$

$\Rightarrow k = \pm \frac{1}{\sqrt{3}}$

$\therefore k = \frac{1}{\sqrt{3}} (\because k > 0)$

Question34

The area of the region A = { (x, y) : 0 ≤ y ≤ x | x | +1 and −1 ≤ x ≤ 1 } in sq. units is:
[Jan. 09, 2019 (II)]

Options:

A. $\frac{2}{3}$

B. 2

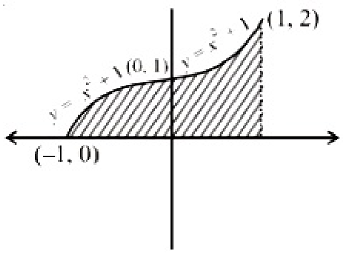
C. $\frac{4}{3}$

D. $\frac{1}{3}$

Answer: B

Solution:

Solution:
Given A = { (x, y) : 0 ≤ y ≤ xx | +1 and −1 ≤ x ≤ 1 }



∴ Area of shaded region

$= \int_{-1}^{0} (-x^2 + 1)dx + \int_{0}^{1} (x^2 + 1)dx$

$= \left(-\frac{x^3}{3} + x\right)_{-1}^{0} + \left(\frac{x^3}{3} + x\right)_{0}^{1}$

$= 0 - \left(\frac{1}{3} - 1\right) + \left(\frac{1}{3} + 1\right) - (0 + 0)$

$= \frac{2}{3} + \frac{4}{3} = \frac{6}{3} = 2$ square units

Question35

The area (in sq. units) of the region A = {(x, y) ∈ R × R | 0d "xd "3, 0d "yd "4, yd "x^2 + 3x} is : [April 8, 2019 (I)]

Options:

A. $\frac{53}{6}$

B. 8

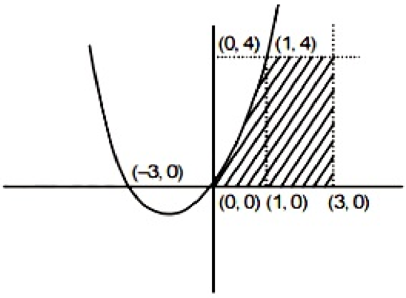
C. $\frac{59}{6}$

D. $\frac{26}{3}$

Answer: C

Solution:

Solution:
Since, the relation $y \le x^2 + 3x$ represents the region below the parabola in the 1^{st} quadrant



$\because y = 4$
$\Rightarrow x^2 + 3x = 4 \Rightarrow x = 1, -4$
$\therefore$ the required area = area of shaded region
$= \int_0^1 (x^2 + 3x)dx + \int_1^3 4 \cdot dx = \left[\frac{x^3}{3} + \frac{3x^2}{2}\right]_0^1 + [4x]_1^3$
$= \frac{1}{3} + \frac{3}{2} + 8 = \frac{59}{6}$

Question36

If the area (in sq. units) of the region $\{(x, y) : y^2 \le 4x, x + y \le 1, x \ge 0, y \ge 0\}$ is $a\sqrt{2} + b$, then $a - b$ is equal to [April 12, 2019 (I)]

Options:

A. $\frac{10}{3}$

B. 6

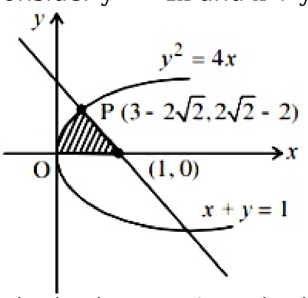
C. $\frac{8}{3}$

D. $-\frac{2}{3}$

Answer: B

Solution:

Solution:
Consider $y^2 = 4x$ and $x + y = 1$



Substituting $x = 1 - y$ in the equation of parabola,
$y^2 = 4(1 - y) \Rightarrow y^2 + 4y - 4 = 0$
$\Rightarrow (y + 2)^2 = 8 \Rightarrow y + 2 = \pm 2\sqrt{2}$
Hence, required area
$= \int_0^{3 - 2\sqrt{2}} 2\sqrt{x}\,dx + \frac{1}{2} \times (2\sqrt{2} - 2) \times (2\sqrt{2} - 2)$
$= \left[2 \times \frac{2}{3}x^{3/2}\right]_0^{3 - 2\sqrt{2}} + \frac{1}{2}(8 + 4 - 8\sqrt{2})$
$\frac{4}{3} \times (3 - 2\sqrt{2})\sqrt{3 - 2\sqrt{2}} + 6 - 4\sqrt{2}$

$= \frac{4}{3}(3 - 2\sqrt{2})(\sqrt{2} - 1) + 6 - 4\sqrt{2}$ $[\because (\sqrt{2} - 1)^2 = 3 - 2\sqrt{2}]$

$= \frac{4}{3}(3\sqrt{2} - 3 - 4 + 2\sqrt{2}) + 6 - 4\sqrt{2}$

$= -\frac{10}{3} + \frac{8}{3}\sqrt{2} = a\sqrt{2} + b$

$\therefore a = 8/3$ and $b = -10/3 \Rightarrow a - b = \frac{10}{3} + \frac{8}{3} = 6$

Question37

If the area (in sq. units) bounded by the parabola $y^2 = 4\lambda x$ and the line $y = \lambda x$, $\lambda > 0$, is $\frac{1}{9}$, then λ is equal to :
[April 12, 2019 (II)]

Options:

A. $2\sqrt{6}$

B. 48

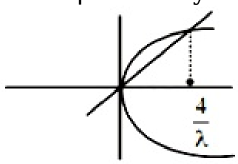
C. 24

D. $4\sqrt{3}$

Answer: C

Solution:

Solution:
Given parabola $y^2 = 4\lambda x$ and the line $y = \lambda x$



Putting $y = \lambda$ in $y^2 = 4\lambda x$, we get $x = 0, \frac{4}{\lambda}$

$\therefore$ required area $= \int_0^{\frac{4}{\lambda}} (2\sqrt{\lambda x} - \lambda x) dx$

$= \left[\frac{2\sqrt{\lambda} \cdot x^{3/2}}{3/2} - \frac{\lambda x^2}{2}\right]_0^{4/\lambda} = \frac{32}{3\lambda} - \frac{8}{\lambda}$

$= \frac{8}{3\lambda} = \frac{1}{9} \Rightarrow \lambda = 24$

Question38

The region represented by $|x - y| \le 2$ and $|x + y| \le 2$ is bounded by a :
[April 10, 2019(I)]

Options:

A. square of side length $2\sqrt{2}$ units

B. rhombus of side length 2 units

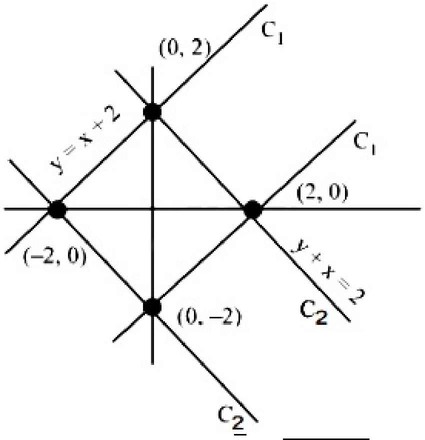
C. square of area 16 sq. units

D. rhombus of area $8\sqrt{2}$ sq. units

Answer: A

Solution:

Solution:
Let, $C_1 : | y - x | \le 2$
$C_2 : | y + x | \le 2$
By the diagram, region is square



Now, length of side $= \sqrt{2^2 + 2^2} = 2\sqrt{2}$

Question39

The area (in sq. units) of the region bounded by the curves $y = 2^x$ and $y = |x + 1|$, in the first quadrant is :
[April 10, 2019(II)]

Options:

A. $\log_e 2 + \frac{3}{2}$

B. $\frac{3}{2}$

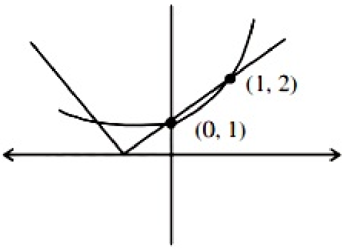
C. $\frac{1}{2}$

D. $\frac{3}{2} - \frac{1}{\log_e 2}$

Answer: D

Solution:

Solution:



Area $= \int_0^1 ((x + 1) - 2^x) dx$ (∵ Area $= \int y dx$)

$= \left[\frac{x^2}{2} + x - \frac{2^x}{\ln 2} \right]_0^1 = \left(\frac{1}{2} + 1 - \frac{2}{\ln 2} \right) - \left(\frac{-1}{\ln 2} \right) = \frac{3}{2} - \frac{1}{\ln 2}$

Question40

The area (in sq. units) of the region $A = \{(x, y) : x^2 \leq y \leq x + 2\}$ is: [April 9, 2019 (I)]

Options:

A. $\frac{10}{3}$

B. $\frac{9}{2}$

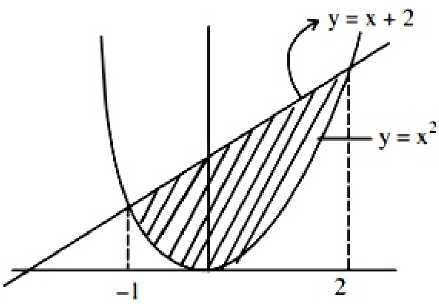
C. $\frac{31}{6}$

D. $\frac{13}{6}$

Answer: B

Solution:

Solution:



Required area is equal to the area under the curves $y \geq x^2$ and yd "x + 2

∴ required area $\int_{-1}^{2} ((x + 2) - x^2) dx$

$= \left(\frac{x^2}{2} + 2x - \frac{x^3}{3} \right)_{-1}^{2} = \left(2 + 4 - \frac{8}{3} \right) - \left(+\frac{1}{2} - 2 + \frac{1}{3} \right) = \frac{9}{2}$

Question41

The area (in sq. units) of the region $A = \left\{ (x, y) : \frac{y^2}{2} \leq x \leq y + 4 \right\}$ is:

[April 09, 2019 (II)]

Options:

A. $\frac{53}{3}$

B. 30

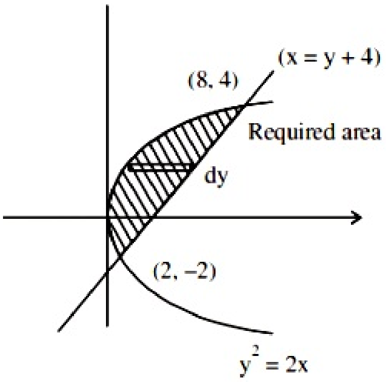
C. 16

D. 18

Answer: D

Solution:

Solution:



Given region, $A = \left\{ (x, y) : \frac{y^2}{2} \le x \le y + 4 \right\}$

Hence, area $= \int_{-2}^{4} x\,dy = \int_{-2}^{4} \left(y + 4 - \frac{y^2}{2} \right) dy$

$= \left[\frac{y^2}{2} + 4y - \frac{y^3}{6} \right]_{-2}^{4} = \left(8 + 16 - \frac{64}{6} \right) - \left(2 - 8 + \frac{8}{6} \right)$

$= \left(24 - \frac{32}{3} \right) - \left(-6 + \frac{4}{3} \right) = \frac{40}{3} + \frac{14}{3} = \frac{54}{3} = 18$

Question42

Let $S(\alpha) = \{(x, y) : y^2 \le x, 0 \le x \le \alpha\}$ and $A(\alpha)$ is area of the region $S(\alpha)$. If for aλ, $0 < \alpha < 4$, $A(\lambda) : A(\alpha) = 2 : 5$, then λequals:
[April 08, 2019 (II)]

Options:

A. $2\left(\frac{4}{25}\right)^{\frac{1}{3}}$

B. $2\left(\frac{2}{5}\right)^{\frac{1}{3}}$

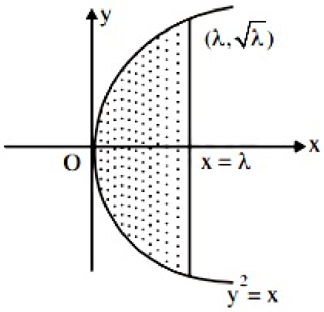
C. $4\left(\frac{2}{5}\right)^{\frac{1}{3}}$

D. $4\left(\frac{4}{25}\right)^{\frac{1}{3}}$

Answer: D

Solution:

Solution:



Area of the region $= 2 \times \int_0^\lambda y\,dx = 2\int_0^\lambda \sqrt{x}\,dx$

$= 2 \times \frac{2}{3}\pi^{\frac{3}{2}}$

$A(\lambda) = 2 \times \frac{2}{3}(\lambda \times \sqrt{\lambda}) = \frac{4}{3}\lambda^{\frac{3}{2}}$

Given, $\frac{A(\lambda)}{A(4)} = \frac{2}{5} \Rightarrow \frac{\lambda^{\frac{3}{2}}}{8} = \frac{2}{5}$

$\lambda = \left(\frac{16}{5}\right)^{\frac{2}{3}} = 4 \cdot \left(\frac{4}{25}\right)^{\frac{1}{3}}$

Question43

Let $g(x) = \cos x^2$, $f(x) = \sqrt{x}$, and alpha, $\beta(\alpha < \beta)$ be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (in sq. units) bounded by the curve $y = (gof)(x)$ and the lines $x = \alpha$, $x = \beta$ and $y = 0$, is :
[2018]

Options:

A. $\frac{1}{2}(\sqrt{3} + 1)$

B. $\frac{1}{2}(\sqrt{3} - \sqrt{2})$

C. $\frac{1}{2}(\sqrt{2} - 1)$

D. $\frac{1}{2}(\sqrt{3} - 1)$

Answer: D

Solution:

Here, $18x^2 - 9\pi x + \pi^2 = 0$
$\Rightarrow (3x - \pi)(6x - \pi) = 0$
$\Rightarrow \alpha = \frac{\pi}{6}, \beta = \frac{\pi}{3}$
Also, gof (x) $= \cos x$
$\therefore$ Req. area $= \int_{\pi/6}^{\pi/3} \cos x \, d x = \frac{\sqrt{3} - 1}{2}$

Question44

If the area of the region bounded by the curves, $y = x^2, y = \frac{1}{x}$ and the lines $y = 0$ and $x = t(t > 1)$ is 1sq. unit, then t is equal to [Online April 16, 2018]

Options:

A. $\frac{4}{3}$

B. $e^{2/3}$

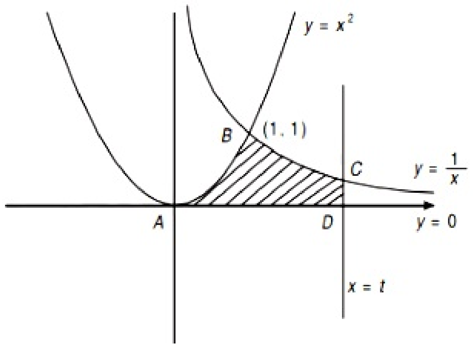
C. $\frac{3}{2}$

D. $e^{3/2}$

Answer: B

Solution:

Solution:
The intersection point of $y = x^2$ and $y = \frac{1}{x}$ is (1,1)
Area bounded by the curves is the region ABCDA is given as:
Area $= \int_0^1 x^2 d x + \int_1^t \frac{1}{x} d x$
$= \left[\frac{x^3}{3}\right]_0^1 + [\ln(x)]_1^t = \frac{1}{3} + \ln(t)$



$\because$ area $= 1$
$\Rightarrow \frac{1}{3} + \ln(t) = 1 \Rightarrow \ln(t) = \frac{2}{3} \Rightarrow t = e^{\frac{2}{3}}$

Question45

$y \leq \sqrt{x}$ }, is
[Online April 15, 2018]

Options:

A. $\frac{13}{3}$

B. $\frac{10}{3}$

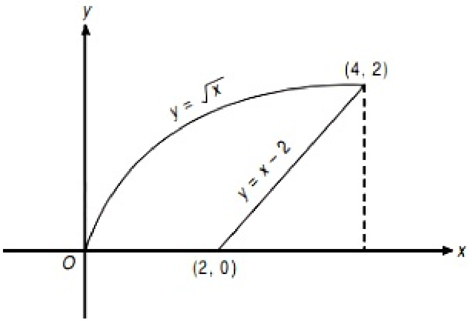
C. $\frac{5}{3}$

D. $\frac{8}{3}$

Answer: B

Solution:

Solution:
The intersection point of $y = x - 2$ and $y = \sqrt{x}$ is (4,2) .
The required area $= \int_0^4 \sqrt{x} d x - \frac{1}{2} \times 2 \times 2 \frac{16}{3} - 2 = \frac{10}{3}$



Question46

The area (in sq. units) of the region { (x, y) : x ≥ 0, x + y ≤ 3, $x^2 \leq 4y$ and $y \leq 1 + \sqrt{x}$ } is
[2017]

Options:

A. $\frac{5}{2}$

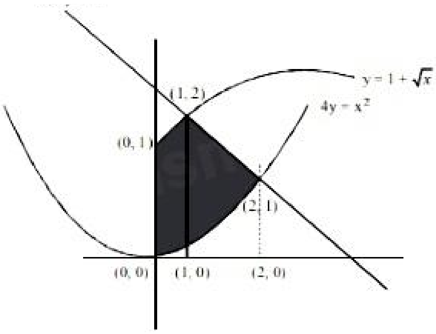
B. $\frac{59}{12}$

C. $\frac{3}{2}$

D. $\frac{7}{3}$

Answer: A

Solution:



Area of shaded region = $\int_0^1 (1+\sqrt{x})\,dx + \int_1^2 (3-x)\,dx - \int_0^2 \frac{x^2}{4}\,dx$

$= [x]_0^1 + \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 + [3x]_1^2 - \left[\frac{x^2}{2}\right]_1^2 - \left[\frac{x^3}{12}\right]_0^2 = \frac{5}{2}$ sq.units

---

Question47

The area (in sq. units) of the smaller portion enclosed between the curves, $x^2 + y^2 = 4$ and $y^2 = 3x$, is:
[Online April 8, 2017]

Options:

A. $\frac{1}{2\sqrt{3}} + \frac{\pi}{3}$

B. $\frac{1}{\sqrt{3}} + \frac{2\pi}{3}$

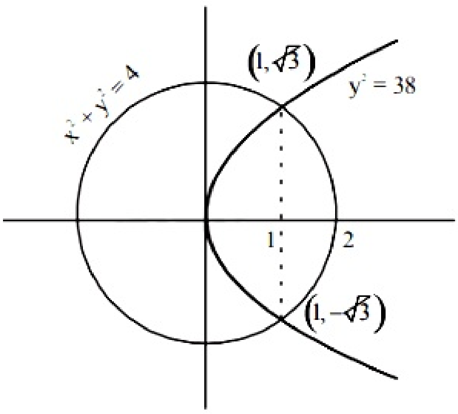
C. $\frac{1}{2\sqrt{3}} + \frac{2\pi}{3}$

D. $\frac{1}{\sqrt{3}} + \frac{4\pi}{3}$

Answer: D

Solution:

Solution:



From the equations we get;
$x^2 + 3x - 4 = 0$
$\Rightarrow (x + 4)(x - 1) = 0 \Rightarrow x = -4, x = 1$

Area $= \int_0^1 \left(\int_0^1 \sqrt{3} \cdot \sqrt{x} d x + \int_1^2 \sqrt{4 - x^2} \cdot d x \right) \times 2$

$= \left(\sqrt{3} \left(\frac{x^{3/2}}{3/2} \right)_0^1 + \left(\frac{x}{2} \sqrt{4 - x^2} + 2\sin^{-1}\frac{x}{2} \right)_1^2 \right) \times 2$

$= \left(\sqrt{3} \left(\frac{2}{3} \right) + \left\{ 2 \cdot \frac{\pi}{2} - \left(\frac{\sqrt{3}}{2} + \frac{\pi}{3} \right) \right\} \right) \times 2$

$= \left(\frac{2}{\sqrt{3}} - \frac{\sqrt{3}}{2} + \frac{2\pi}{3} \right) \times 2$

$= \left(\frac{1}{2\sqrt{3}} + \frac{2\pi}{3} \right) \times 2 = \frac{1}{\sqrt{3}} + \frac{4\pi}{3}$

Question48

The area (in sq. units) of the region $\{ (x, y) : y^2 \geq 2x$ and $x^2 + y^2 \leq 4x, x \geq 0, y \geq 0 \}$ is:
[2016]

Options:

A. $\pi - \frac{4\sqrt{2}}{3}$

B. $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$

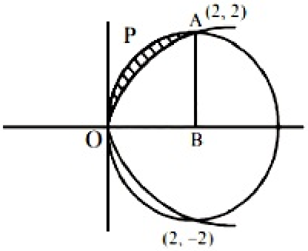
C. $\pi - \frac{4}{3}$

D. $\pi - \frac{8}{3}$

Answer: D

Solution:

Solution:



Points of intersection of the two curves are (0,0),(2,2) and (2,-2)
Area = Area (OPAB) − area under parabola (0 to 2)

$= \frac{\pi \times (2)^2}{4} - \int_0^2 \sqrt{2}\sqrt{x} d x = \pi - \frac{8}{3}$

Question49

The area (in sq. units) of the region described by $A = \{ (x, y) \mid y \geq x^2 - 5x + 4, x + y \geq 1, y \leq 0 \}$ is:
[Online April 9, 2016]

A. $\frac{19}{6}$

B. $\frac{17}{6}$

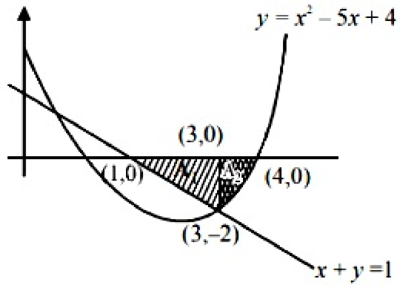
C. $\frac{7}{2}$

D. $\frac{13}{6}$

Answer: A

Solution:

Solution:



Required area $= A_1 + A_2$

$= \frac{1}{2} \times 2 \times 2 + \left| \int_3^4 (x^2 - 5x + 4) dx \right|$

$= 2 + \frac{7}{6} = \frac{19}{6}$ sq. units

Question50

The area (in sq. units) of the region described by { (x, y) : $y^2 \leq 2x$ and $y \geq 4x - 1$ } is
[2015]

Options:

A. $\frac{15}{64}$

B. $\frac{9}{32}$

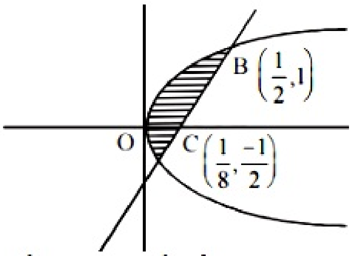
C. $\frac{7}{32}$

D. $\frac{5}{64}$

Answer: B

Solution:

Solution:
Required area



$= \int_{-1/2}^{1} \frac{y+1}{4} dy - \int_{-1/2}^{1} \frac{y^2}{2} dy$

$= \frac{1}{4}\left[\frac{y^2}{2}+y\right]_{-1/2}^{1} - \frac{1}{2}\left[\frac{y^3}{3}\right]_{-1/2}^{1}$

$= \frac{1}{4}\left[\frac{3}{2}+\frac{3}{8}\right] - \frac{9}{48} = \frac{15}{32} - \frac{9}{48} = \frac{27}{96} = \frac{9}{32}$

Question51

The area (in square units) of the region bounded by the curves $y + 2x^2 = 0$ and $y + 3x^2 = 1$, is equal to
[Online April 10, 2015]

Options:

A. $\frac{3}{5}$

B. $\frac{1}{3}$

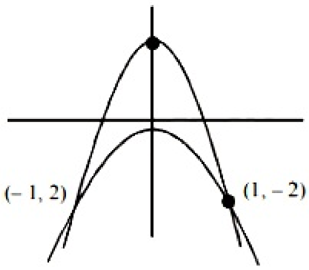
C. $\frac{4}{3}$

D. $\frac{3}{4}$

Answer: C

Solution:

Solution:
Solving
$y + 2x^2 = 0$
$y + 3x^2 = 1$



Point of intersection (1,-2) and (-1,-2)

Area $= 2\int_0^1 ((1-3x^2) - (-2x^2)) dx$

$2\int_0^1 (1-x^2) dx = 2\left(x - \frac{x^3}{3}\right)_0^1 = \frac{4}{3}$

$= 15 - 6 = 9$ sq units

Question52

The area of the region described by $A = \{ (x, y) : x^2 + y^2 \leq 1$ and $y^2 \leq 1 - x \}$ is:
[2014]

Options:

A. $\frac{\pi}{2} - \frac{2}{3}$

B. $\frac{\pi}{2} + \frac{2}{3}$

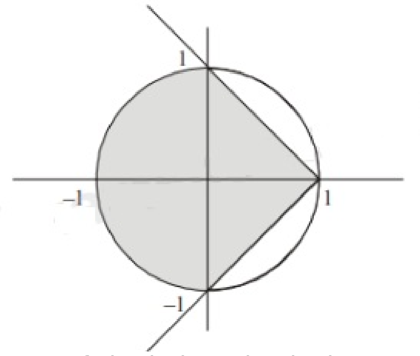
C. $\frac{\pi}{2} + \frac{4}{3}$

D. $\frac{\pi}{2} - \frac{4}{3}$

Answer: C

Solution:

Solution:
Given curves are $x^2 + y^2 = 1$ and $y^2 = 1 - x$.
Intersecting points are $x = 0, 1$



Area of shaded portion is the required area.
So, Required Area = Area of semi-circle + Area bounded by parabola

$= \frac{\pi r^2}{2} + 2\int_0^1 \sqrt{1-x}\,dx = \frac{\pi}{2} + 2\int_0^1 \sqrt{1-x}\,dx$ ($\because$ radius of circle $= 1$)

$= \frac{\pi}{2} + 2\left[\frac{(1-x)^{3/2}}{-3/2}\right]_0^1 = \frac{\pi}{2} - \frac{4}{3}(-1) = \frac{\pi}{2} + \frac{4}{3}$ sq. unit

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Question53

The area of the region above the x -axis bounded by the curve $y = \tan x,\ 0 \leq x \leq \frac{\pi}{2}$ and the tangent to the curve at$x = \frac{\pi}{4}$ is:
[Online April 19, 2014]

Options:

A. $\frac{1}{2}\left(\log 2 - \frac{1}{2}\right)$

B. $\frac{1}{2}\left(\log 2 + \frac{1}{2}\right)$

C. $\frac{1}{2}(1 - \log 2)$

D. $\frac{1}{2}(1 + \log 2)$

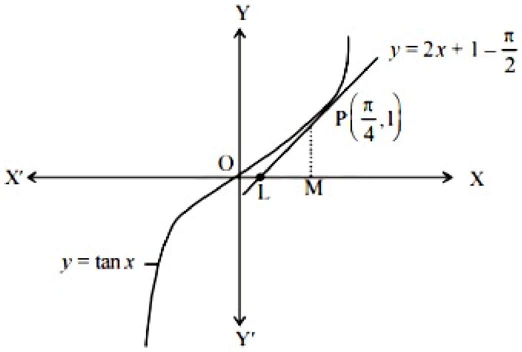
Answer: A

Solution:

Solution:
The given curve is $y = \tan x$ ....(1)

when $x = \frac{\pi}{4}, y = 1$

Equation of tangent at P is $y - 1 = \left(\sec^2\frac{\pi}{4}\right)\left(x - \frac{\pi}{4}\right)$



or $y = 2x + 1 - \frac{\pi}{2}$ ....(2)

Area of shaded region
= area of OPM O − ar(ΔPLM)

$= \int_0^{\frac{\pi}{4}} \tan x \, d\, x - \frac{1}{2}(OM - OL)PM$

$= [\log \sec x]_0^{\frac{\pi}{4}} - \frac{1}{2}\left\{\frac{\pi}{4} - \frac{\pi - 2}{4}\right\} \times 1$

$= \frac{1}{2}\left[\log 2 - \frac{1}{2}\right]$ sq unit

Question54

Let $A = \{(x, y) : y^2 \leq 4x, y - 2x \geq -4\}$. The area (in square units) of the region A is:
[Online April 9, 2014]

Options:

A. 8

B. 9

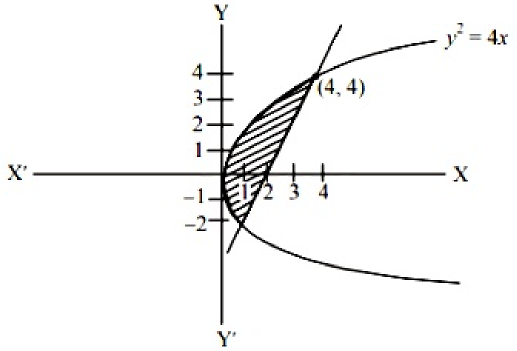
C. 10

D. 11

Answer: B

Solution:

Solution:
Area of shaded portion



$= \left| \int_{2}^{4} \left(\frac{y+4}{2} \right) dy \right| - \left| \int_{-2}^{4} \frac{y^2}{4} dy \right|$

$= \left| \frac{1}{2} \left[\frac{y^2}{2} + 4y \right]_{-2}^{4} \right| - \left| \frac{1}{4} \left[\frac{y^3}{3} \right]_{-2}^{4} \right|$

$= \left| \frac{1}{2} [\{8 + 16\} - \{2 - 8\}] \right| - \left| \frac{1}{4} \left\{ \frac{64}{3} + \frac{8}{3} \right\} \right| = 9$

Question55

Let $f : [-2, 3] \to [0, \infty)$ be a continuous function such that $f(1 - x) = f(x)$ for all $x \in [-2, 3]$.
If R_1 is the numerical value of the area of the region bounded by $y = f(x)$, $x = -2$, $x = 3$ and the axis of x and $R_2 = \int_{-2}^{3} xf(x)dx$, then :
[Online April 25, 2013]

Options:

A. $3R_1 = 2R_2$

B. $2R_1 = 3R_2$

C. $R_1 = R_2$

D. $R_1 = 2R_2$

Answer: D

Solution:

Solution:
We have

$R_2 = \int_{-2}^{3} xf(x)dx = \int_{-2}^{3} (1 - x)f(1 - x)dx$ Using $\int_a^b f(x)dx = \int_a^b f(a + b - x)dx \Big]$

$\Rightarrow R_2 = \int_{-2}^{3} (1 - x)f(x)dx$ ($\because f(x) = f(1 - x)$ on $[-2, 3]$)

$\therefore R_2 + R_2 = \int_{-2}^{3} xf(x)dx + \int_{-2}^{3} (1 - x)f(x)dx = \int_{-2}^{3} f(x)dx = R_1$

$\Rightarrow R_1 = 2R_2$

The area (in square units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, x -axis, and lying in the first quadrant is:
[2013]

Options:

A. 9

B. 36

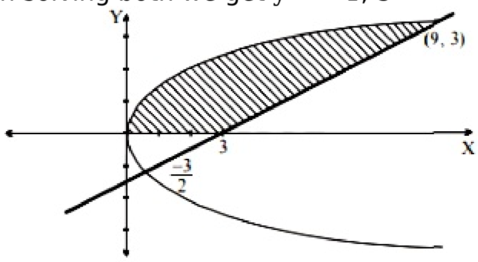
C. 18

D. $\frac{27}{4}$

Answer: A

Solution:

Solution:
Given curves are
$y = \sqrt{x}$ .....(1)
and $2y - x + 3 = 0$ ....(2)
On solving both we get $y = -1, 3$



Required area $= \int_0^3 \{(2y + 3) - y^2\} dy$

$= \left[y^2 + 3y - \frac{y^3}{3} \right]_0^3 = 9$

Question57

The area under the curve $y = |\cos x - \sin x|$, $0 \le x \le \frac{\pi}{2}$, and above x -axis is :
[Online April 23, 2013]

Options:

A. $2\sqrt{2}$

B. $2\sqrt{2} - 2$

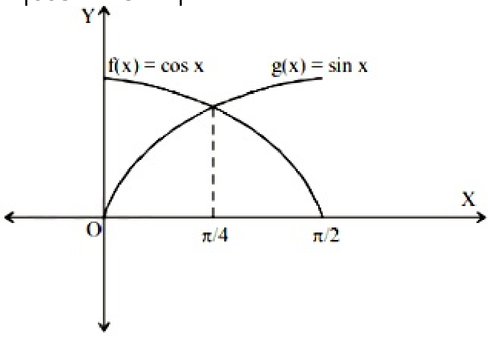
C. $2\sqrt{2} + 2$

D. 0

Answer: B

Solution:

Solution:
$y = |\cos x - \sin x|$



Required area $= 2\int_{0}^{\pi/4}(\cos x - \sin x)dx$
$= 2[\sin x + \cos x]_0^{\pi/4}$
$= 2\left[\frac{2}{\sqrt{2}} - 1\right] = (2\sqrt{2} - 2)$ sq. units

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Question58

The area of the region (in sq. units), in the first quadrant bounded by the parabola $y = 9x^2$ and the lines x = 0, y = 1 and y = 4, is:
[Online April 22, 2013]

Options:

A. 7/9

B. 14/3

C. 7/3

D. 14/9

Answer: D

Solution:

Solution:
Required area $= \int_{y=1}^{4}\sqrt{\frac{y}{9}}dy$
$= \frac{1}{3}\int_{y=1}^{4} y^{1/2}dy = \frac{1}{3} \times \frac{2}{3}(y^{3/2})|^4$
$= \frac{2}{9}[(4^{1/2})^3 - (1^{1/2})^3] = \frac{2}{9}[8 - 1]$
$= \frac{2}{9} \times 7 = \frac{14}{9}$sq. units.

Question59

The area bounded by the curve y = ln(x) and the lines y = 0, y = ln(c) and x = 0 is equal to :
[Online April 9, 2013]

A. 3

B. $3\ln(c) - 2$

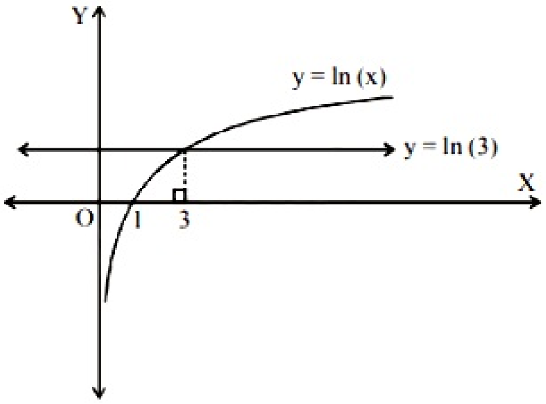
C. $3\ln(c) + 2$

D. 2

Answer: D

Solution:

To find the point of intersection of curves $y = \ln(x)$ and $y = \ln(3)$, put $\ln(x) = \ln(3)$
$\Rightarrow \ln(x) - \ln(3) = 0$
$\Rightarrow \ln(x) - \ln(3) = \ln(1)$
$\Rightarrow \frac{x}{3} = 1, \Rightarrow x = 3$



Required area $= \int_0^3 \ln(3)\,dx - \int_1^3 \ln(x)\,dx$
$= [x\ln(3)]_0^3 - [x\ln(x) - x]_1^3 = 2$

Question60

The area between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$ and the straight line $y = 2$ is:
[2012]

Options:

A. $20\sqrt{2}$

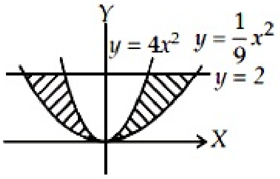
B. $\frac{10\sqrt{2}}{3}$

C. $\frac{20\sqrt{2}}{3}$

D. $10\sqrt{2}$

Answer: C

Solution:



Required area $= 2\int_0^2 \left(\sqrt{9y} - \sqrt{\frac{y}{4}}\right) dy$

$= 2\int_0^2 \left(3\sqrt{y} - \frac{\sqrt{y}}{2}\right) dy = 2\left[\frac{2}{3} \times 3 \cdot y^{\frac{3}{2}} - \frac{1}{2} \times \frac{2}{3} \cdot y^{\frac{3}{2}}\right]_0^2$

$= 2\left[2y^{\frac{3}{2}} - \frac{1}{3}y^{\frac{3}{2}}\right]_0^2 = 2 \times \left[\frac{5}{3}y^{\frac{3}{2}}\right]_0^2$

$= 2 \cdot \frac{5}{3} 2\sqrt{2} = \frac{20\sqrt{2}}{3}$

Question61

The area bounded by the parabola $y^2 = 4x$ and the line $2x - 3y + 4 = 0$, in square unit, is
[Online May 26, 2012]

Options:

A. $\frac{2}{5}$

B. $\frac{1}{3}$

C. 1

D. $\frac{1}{2}$

Answer: B

Solution:

Solution:
Intersecting points are $x = 1, 4$

$\therefore$ Required area $= \int_1^4 \left[2\sqrt{x} - \left(\frac{2x+4}{3}\right)\right] dx$

$= \left.\frac{2x^{3/2}}{3/2}\right|_1^4 - \left.\frac{2x^2}{3 \times 2}\right|_1^4 - \left.\frac{4}{3}x\right|_1^4$

$= \frac{4}{3}(4^{3/2} - 1^{3/2}) - \frac{1}{3}(16 - 1) - \left[\frac{4}{3}(4) - \frac{4}{3}\right]$

$= \frac{4}{3}(7) - 5 - 4 = \frac{28}{3} - 9 = \frac{28 - 27}{3} = \frac{1}{3}$

Question62

The area of the region bounded by the curve $y = x^3$, and the lines, $y = 8$, and $x = 0$, is
[Online May 19, 2012]

Options:

A. 8

B. 12

C. 10

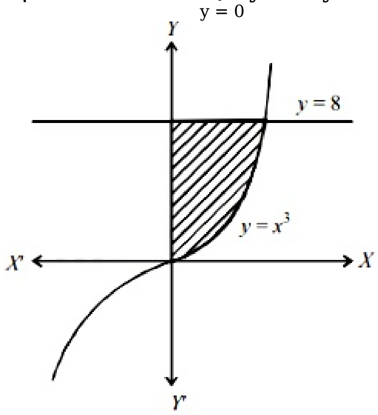
D. 16

Answer: B

Solution:

Solution:

Required Area $= \int_{y=0}^{8} y^{1/3} dy$



$= \left.\frac{y^{1/3+1}}{\frac{1}{3}+1}\right|_0^8 = \frac{3}{4}(y^{4/3})\Big|_0^8$

$= \frac{3}{4}\left[(8)^{\frac{4}{3}} - 0\right] = \frac{3}{4}[2^4] = \frac{3}{4} \times 16 = 12$sq. unit

----

Question63

If a straight line $y - x = 2$ divides the region $x^2 + y^2 \le 4$ into two parts, then the ratio of the area of the smaller part to the area of the greater part is
[Online May 12, 2012]

Options:

A. $3\pi - 8 : \pi + 8$

B. $\pi - 3 : 3\pi + 3$

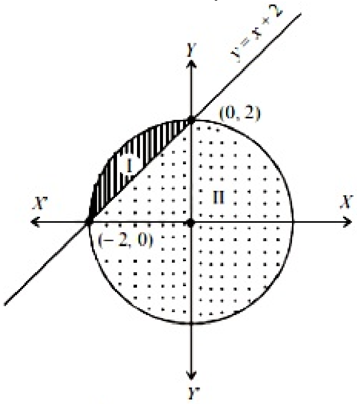
C. $3\pi - 4 : \pi + 4$

D. $\pi - 2 : 3\pi + 2$

Answer: D

Solution:

Let I be the smaller portion and II be the greater portion of the given figure then,



Area of I $= \int_{-2}^{0} [\sqrt{4-x^2} - (x+2)]dx$

$= \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_{-2}^{0} - \left[\frac{x^2}{2} + 2x\right]_{-2}^{0}$

$= [2\sin^{-1}(-1)] - \left[-\frac{4}{2} + 4\right] = 2 \times \frac{\pi}{2} - 2 = \pi - 2$

Now, area of II = Area of circle-area of I. $= 4\pi - (\pi - 2) = 3\pi + 2$

Hence, required ratio $= \frac{\text{area of I}}{\text{area of II}} = \frac{\pi - 2}{3\pi + 2}$

Question64

The area enclosed by the curves $y = x^2$, $y = x^3$, $x = 0$ and $x = p$, where $p > 1$, is 1 / 6. The p equals
[Online May 12, 2012]

Options:

A. 8/3

B. 16/3

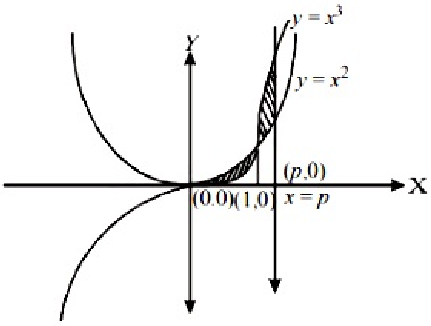
C. 2

D. 4/3

Answer: D

Solution:

Solution:
(d) Given curves are $y = x^2$ and $y = x^3$
Also, $x = 0$ and $x = p$, $p > 1$
Now, intersecting point is (1,1)



$$\frac{1}{6} = \frac{x^3}{3} - \frac{x^4}{4}\Big|_0^1 + \frac{x^4}{4} - \frac{x^3}{3}\Big|_1^p$$

$$\Rightarrow \frac{1}{6} = \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{p^4}{4} - \frac{p^3}{3} - \frac{1}{4} + \frac{1}{3}\right)$$

$$\Rightarrow \frac{1}{6} - \frac{1}{3} + \frac{1}{4} + \frac{1}{4} - \frac{1}{3} = \frac{3p^4 - 4p^3}{12}$$

$$\Rightarrow \frac{p^3(3p-4)}{12} = 0 \Rightarrow p^3(3p-4) = 0$$

$$\Rightarrow p = 0 \text{ or } \frac{4}{3}$$

Since, it is given that $p > 1$

$\therefore p$ can not be zero.

Hence, $p = \frac{4}{3}$

Question65

The parabola $y^2 = x$ divides the circle $x^2 + y^2 = 2$ into two parts whose areas are in the ratio
[Online May 7, 2012]

Options:

A. $9\pi + 2 : 3\pi - 2$

B. $9\pi - 2 : 3\pi + 2$

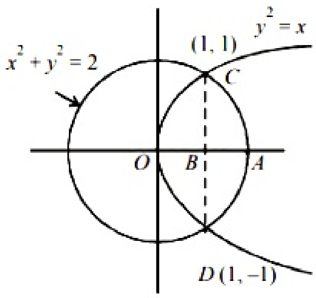
C. $7\pi i - 2 : 2\pi - 3$

D. $7\pi + 2 : 3\pi + 2$

Answer: B

Solution:

Solution:



Area of circle $= \pi(\sqrt{2})^2 = 2\pi$

Area of OCADO = 2{Area of OCAO}

= 2 {area of OCB + area of BCA}

$$= 2\int_0^1 y_p\,dx + 2\int_1^{\sqrt{2}} y_c\,dx$$

where $y_p = \sqrt{x}$ and $y_c = \sqrt{2 - x^2}$

$$\therefore \text{Required Area } = 2\int_0^1 \sqrt{x}\,dx + 2\int_1^{\sqrt{2}} \sqrt{2 - x^2}\,dx$$

$$= 2\left[\frac{2}{3}.1 - 0\right] + 2\left[\frac{x\sqrt{2 - x^2}}{2} + \sin^{-1}\frac{x}{\sqrt{2}}\right]_1^{\sqrt{2}}$$

$$= \frac{4}{3} + 2\left\{\frac{\pi}{2} - \frac{\pi}{4} - \frac{1}{2}\right\} = \frac{4}{3} + 2\left\{\frac{\pi}{4} - \frac{1}{2}\right\} = \frac{3\pi + 2}{6}$$

∴ Required Ratio $= \frac{9\pi - 2}{3\pi + 2}$ i.e., $9\pi - 2 : 3\pi + 2$

Question66

The area bounded by the curves $y^2 = 4x$ and $x^2 = 4y$ is: [2011 RS]

Options:

A. $\frac{32}{3}$sq units

B. $\frac{16}{3}$sq units

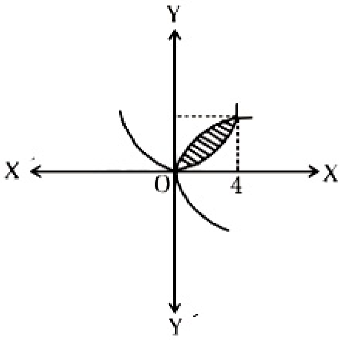
C. $\frac{8}{3}$sq. units

D. 0sq. units

Answer: B

Solution:

Solution:



Required area $= \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right) dx$

$= \left[2\left(\frac{x^{3/2}}{3/2}\right) - \frac{x^3}{12}\right]_0^4$

$= \frac{4}{3} \times 8 - \frac{64}{12} = \frac{32}{3} - 16 = \frac{16}{3}$ sq. units

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Question67

The area of the region enclosed by the curves $y = x$, $x = e$, $y = \frac{1}{x}$ and the positive x -axis is [2011]

Options:

A. 1 square unit

B. $\frac{3}{2}$ square units

C. $\frac{5}{2}$ square units

D. $\frac{1}{2}$ square unit

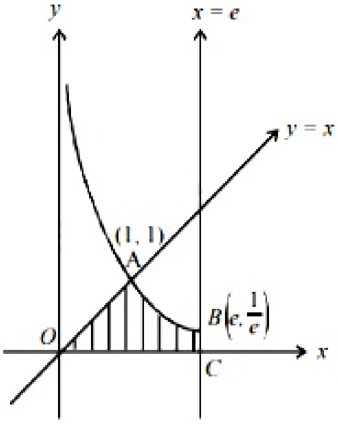
Answer: B

Solution:

Area of required region AOCBO

$= \int_0^1 x\,dx + \int_1^e \frac{1}{x}dx = \left[\frac{x^2}{2}\right]_0^1 + [\log x]_1^e$

$= \frac{1}{2} + 1 = \frac{3}{2}$ sq. units



Question68

The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$ is
[2010]

Options:

A. $4\sqrt{2} + 2$

B. $4\sqrt{2} - 1$

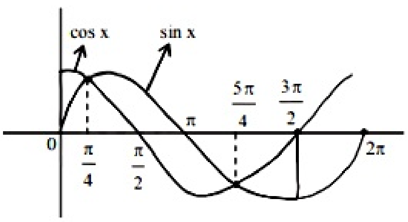
C. $4\sqrt{2} + 1$

D. $4\sqrt{2} - 2$

Answer: D

Solution:

Solution:



Area above x -axis = Area below x -axis

$\therefore$ Required area $= 2\left[\int_0^{\pi/4}(\cos x - \sin x)dx + \int_{\pi/4}^{\pi}\sin x\,dx - \int_{\pi/4}^{\pi/2}\cos x\,dx\right]$

$= 2[(\sin x + \cos x)_0^{\pi/4} + (-\cos x)_{\pi/4}^{\pi} - (\sin x)_{\frac{\pi}{4}}^{\frac{\pi}{2}}]$

$= 2\left[\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - (0+1) + \left(1 + \frac{1}{\sqrt{2}}\right) - \left(1 - \frac{1}{\sqrt{2}}\right)\right]$

$= 2\left[\sqrt{2} - 1 + 1 + \frac{1}{\sqrt{2}} - 1 + \frac{1}{\sqrt{2}}\right]$

$= 2[\sqrt{2} + \sqrt{2} - 1] = 4\sqrt{2} - 2$

Question69

The area of the region bounded by the parabola $(y-2)^2 = x - 1$, the tangent of the parabola at the point (2,3) and the x -axis is: [2009]

Options:

A. 6

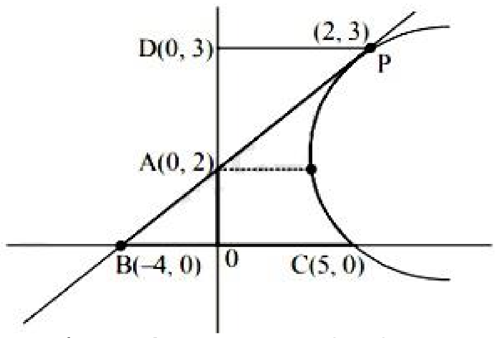
B. 9

C. 12

D. 3

Answer: B

Solution:



For slope of tangents at (2,3)

$(y-2)^2 = x - 1$

$2(y-2)\frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{2(y-2)} m = \left(\frac{dy}{dx}\right)_{(2,3)} = \frac{1}{2(3-2)} = \frac{1}{2}$

Equation of tangent

$y - 3 = \frac{1}{2}(x-2)$

$\Rightarrow x - 2y + 4 = 0$ .....(i)

The given parabola is $(y-2)^2 = x - 1$ ....(ii)

vertex (1,2) and it meets x -axis at (5,0)

Then required area $= Ar\Delta BOA + Ar(OCPD) - Ar(\Delta APD)$

$= \frac{1}{2} \times 4 \times 2 + \int_0^3 x\,dy - \frac{1}{2} \times 2 \times 1$

$= 3 + \int_0^3 (y-2)^2 + 1\,dy = 3 + \left[\frac{(y-2)^3}{3} + y\right]_0^3$

$= 3 + \left[\frac{1}{3} + 3 + \frac{8}{3}\right] = 3 + 6 = 9$sq. units

Question70

The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 =$ lis equal to [2008]

Options:

A. $\frac{5}{3}$

B. $\frac{1}{3}$

C. $\frac{2}{3}$

D. 43

Answer: D

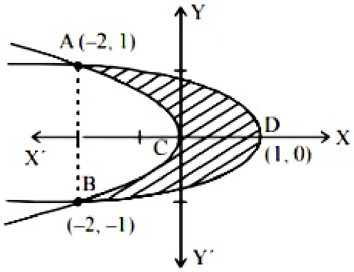
Solution:

Solution:

Given $x + 2y^2 = 0 \Rightarrow y^2 = -\frac{x}{2}$

and $x + 3y^2 = 1 \Rightarrow y^2 = -\frac{1}{3}(x - 1)$

On solving these two equations we get the points of intersection as (-2,1),(-2,-1)



The required area is ACBDA, given by

$$A = 2\left\{\int_{-2}^{1}\frac{1}{\sqrt{3}}\sqrt{1-x}\,dx - \frac{1}{\sqrt{2}}\int_{-2}^{0}\sqrt{-x}\,dx\right\}$$

$$\Rightarrow 2\left\{\frac{1}{\sqrt{3}}\left[\frac{2}{3}(1-x)^{3/2}\right]_{-2}^{1} - \frac{1}{\sqrt{2}}\left[\frac{2}{3}(-x)^{3/2}\right]_{-2}^{0}\right\}$$

$$\Rightarrow 2\left\{\left[-\frac{1}{\sqrt{3}}\times\frac{2}{3}(0-3^{3/2})\right] - \left[\frac{-1}{\sqrt{2}}\times\frac{2}{3}(0-2^{3/2})\right]\right\}\}$$

$$\Rightarrow 2\left\{\frac{2}{3\sqrt{3}}\times 3\sqrt{3} - \frac{1}{\sqrt{2}}\times\frac{2}{3}.2\sqrt{2}\right\}$$

$$\Rightarrow 2\left\{2-\frac{4}{3}\right\} = 2\left\{\frac{6-4}{3}\right\} = \frac{4}{3}\text{sq. units}$$

Question71

The area enclosed between the curves $y^2 = x$ and $y = |x|$ is [2007]

Options:

A. 1/6

B. 1/3

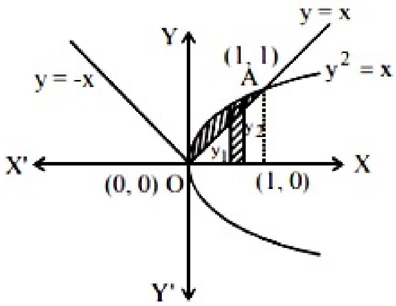
C. 2/3

D. 1

Answer: A

Solution:

It is clear from the figure, area lies between $y^2 = x$ and $y = x$
Intersection point $y = x$ and $y^2 = x$ is (1,1)



$\therefore$ Required area $= \int_0^1 (y_2 - y_1)dx$

$= \int_0^1 (\sqrt{x} - x)dx = \left[\frac{x^{3/2}}{3/2} - \frac{x^2}{2}\right]_0^1$

$= \frac{2}{3}[x^{3/2}]_0^1 - \frac{1}{2}[x^2]_0^1 = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$

Question72

Let f (x) be a non − negative continuous function such that the area bounded by the curve y = f (x), x -axis and the ordinates $x = \frac{\pi}{4}$ and $x = \beta > \frac{\pi}{4}$ is

$\left(\beta \sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\beta\right)$. Then $f\left(\frac{\pi}{2}\right)$ is
[2005]

Options:

A. $\left(\frac{\pi}{4} + \sqrt{2} - 1\right)$

B. $\left(\frac{\pi}{4} - \sqrt{2} + 1\right)$

C. $\left(1 - \frac{\pi}{4} - \sqrt{2}\right)$

D. $\left(1 - \frac{\pi}{4} + \sqrt{2}\right)$

Answer: D

Solution:
From given condition
$\int_{\pi/4}^{\beta} f(x)dx = \beta \sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\beta$
Differentiating w. r . tβ, we get
$f(\beta) = \beta\cos\beta + \sin\beta - \frac{\pi}{4}\sin\beta + \sqrt{2}$
$f\left(\frac{\pi}{2}\right) = \beta . 0 + \left(1 - \frac{\pi}{4}\right)\sin\frac{\pi}{2} + \sqrt{2} = 1 - \frac{\pi}{4} + \sqrt{2}$

Question73

The area enclosed between the curve $y = \log_e(x + e)$ and the coordinate axes is
[2005]

Options:

A. 1

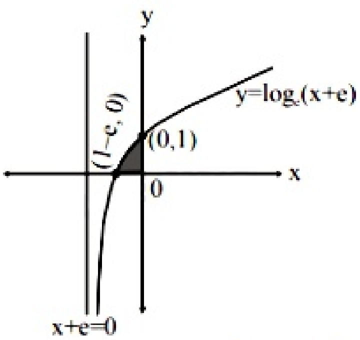
B. 2

C. 3

D. 4

Answer: A

Solution:



Required area $A = \int_{1-e}^{0} y\,dx = \int_{1-e}^{0} \log_e(x+e)dx$
put $x + e = t \Rightarrow dx = dt$ also when $x = 1 - e$, $t = 1$ and when $x = 0$, $t = e$
$\therefore A = \int_1^e \log_e t\,dt = [t\log_e t - t]_1^e$
$e - e - 0 + 1 = 1$
Hence the required area is 1 square unit.

Question74

The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines $x = 4$, $y = 4$ and the coordinate axes. If S_1, S_2, S_3 are respectively the areas of these parts numbered from top to bottom; then

[2005]

Options:

A. 1 : 2 : 1

B. 1 : 2 : 3

C. 2 : 1 : 2

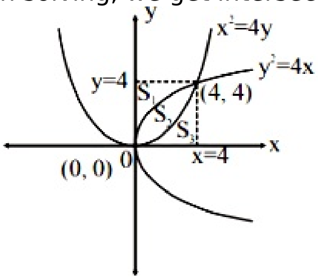
D. 1 : 1 : 1

Answer: D

Solution:

Solution:

On solving, we get intersection points of $x^2 = 4y$ and $y^2 = 4x$ are (0,0) and (4,4)



By symmetry, we observe

$S_1 = S_3 = \int_0^4 y\,dx$

$= \int_0^4 \frac{x^2}{4} dx = \left[\frac{x^3}{12}\right]_0^4 = \frac{16}{3}$ sq. units

Also $S_2 = \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right) dx = \left[\frac{2x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^3}{12}\right]_0^4$

$= \frac{4}{3} \times 8 - \frac{16}{3} = \frac{16}{3}$ sq. units

$\therefore S_1 : S_2 : S_3 = 1 : 1 : 1$

Question75

The area of the region bounded by the curves $y = |x - 2|$, $x = 1$, $x = 3$ and the x -axis is
[2004]

Options:

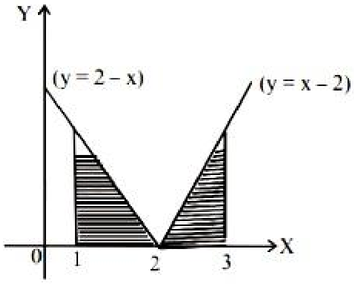
A. 4

B. 2

C. 3

D. 1

Answer: D



Required Area

$A = 2\int_{2}^{3}(x-2)dx = 2\left[\frac{x^2}{2} - 2x\right]_2^3 = 1$

Question76

The area of the region bounded by the curves y = |x − 1| and y = 3− |x| is [2003]

Options:

A. 6 sq. units

B. 2 sq. units

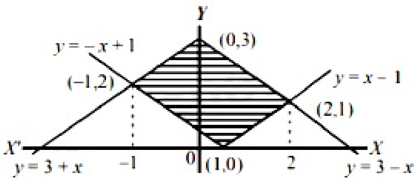
C. 3 sq. units

D. 4 sq. units.

Answer: D

Solution:

Solution:
Intersection point of $y = x - 1$ and $y = 3 - x$ is (2,1) and eqns. $y = -x + 1$ and $y = 3 + x$ is (-1,2)



$A = \int_{-1}^{0}\{(3+x)-(-x+1)\}dx +$

$\int_{0}^{1}\{(3-x)-(-x+1)\}dx + \int_{1}^{2}\{(3-x)-(x-1)\}dx$

$= \int_{-1}^{0}(2+2x)dx + \int_{0}^{1}2dx + \int_{1}^{2}(4-2x)dx$

$= [2x+x^2]_{-1}^{0} + [2x]_0^1 + [4x-x^2]_1^2$

$= 0-(-2+1)+(2-0)+(8-4)-(4-1)$

$= 1+2+4-3 = 4$ sq. units

Question77

encloses an area of 3 / 4 square unit with the axes then $\int_0^2 xf'(x)dx$ is [2002]

Options:

A. 3/2

B. 1

C. 5/4

D. –3/4

Answer: D

Solution:

Solution:

Given that $\int_0^2 f(x)dx = \frac{3}{4}$; Now,

$\int_0^2 xf'(x)dx = x\int_0^2 f'(x)dx - \int_0^2 f(x)dx$

$= [xf(x)]_0^2 - \frac{3}{4} = 2f(2) - \frac{3}{4}$

$= 0 - \frac{3}{4}(\because f(2) = 0) = -\frac{3}{4}$

Question78

The area bounded by the curves $y = \ln x$, $y = \ln|x|$, $y = |\ln x|$ and $y = |\ln|x||$ is [2002]

Options:

A. 4sq. units

B. 6 sq. units

C. 10 sq. units

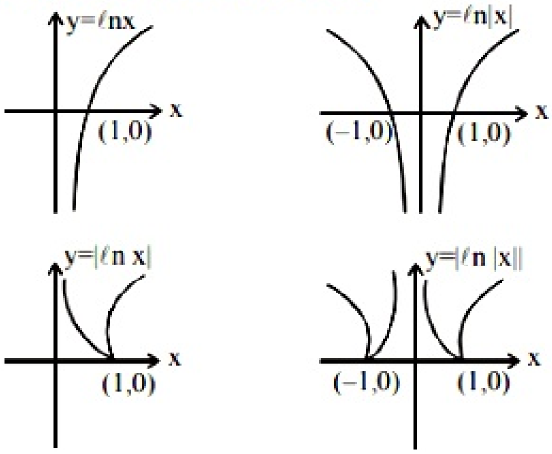
D. none of these

Answer: A

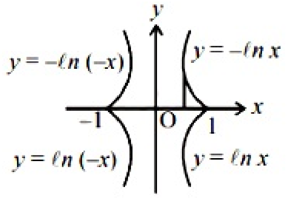
Solution:

Solution:

Separate graph of each curve



[Note: Graph of $y = |f(x)|$ can be obtained from the graph of the curve $y = f(x)$ by drawing the mirror image of the portion of the graph below x -axis, with respect to x -axis.
Hence the bounded area is as shown by combined all figure.



Required area $= 4\int_0^1 (-\ln x)dx$

$= -4[x\ln x - x]_0^1 = 4$sq. units
